Dimension reduction on heterogeneous networks

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The complexity inherent to large, non-linear dynamical systems of interacting units (ecological communities, neuronal assemblies, etc.) makes them very difficult to study. One of the big challenges of network science is finding ways to approximate these systems by systems of reduced dimension, which can be more tractable both analytically and computationally [1, 2, 3, 4]. We have addressed this problem for dynamical systems whose units interact through a weighted, directed connectivity matrix. Following the lines of previous work [3, 4], we propose a two-step method for such a dimension reduction that takes into account the properties of the interaction matrix. First, units are partitioned into groups of similar connectivity properties. Each group is associated to one linear observable, that is a weighted average of the node activities within the group. The number of groups thus defines the dimension of the reduced system of observables. Second, we derive a set of conditions that have to be fulfilled for these observables to properly represent the original system's behavior, together with a method for approximately solving them. The result is a reduced interaction matrix and an approximate system of ODEs for the temporal evolution of the observables which is analogous in form to the original system. We show that the reduced system can be used to predict some properties of the complete dynamics (such as bifurcation boundaries in the parameter space) for different types of connectivity structures, including highly heterogeneous networks.

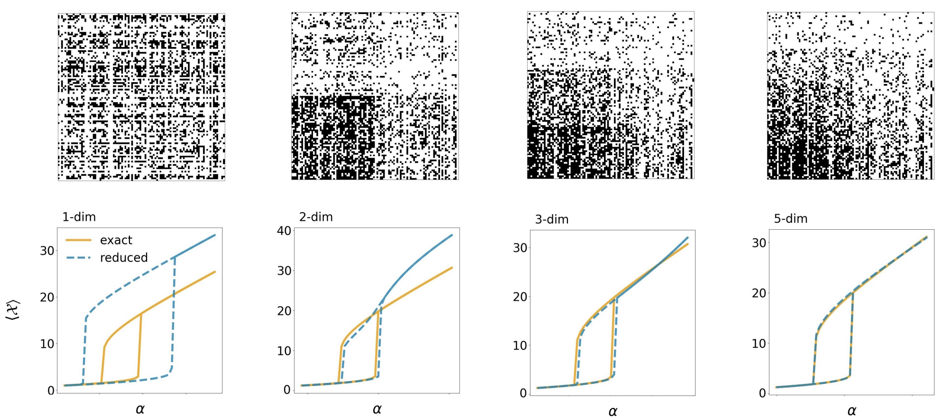

Figure 1: Dimension reduction of a heterogeneous network on 100 nodes with Wilson-Cowan dynamics. From left to right, different node groupings are shown (1, 2, 3, and 5 groups). **Top:** Connectivity matrix, reordered according to the different node groupings. **Bottom:** Bifurcation diagram of the observable dynamics for the different groupings (orange: exact dynamics, blue: approximate, reduced dynamics). The x-axis shows the bifurcation parameter, which controls the overall strength of the connections in the network. The y-axis shows the observables' average at equilibrium.

References

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