Statistical mechanics of mesoscopic structure extraction

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At the mesoscopic level of structures, the nodes and links of networks organize in patterns such as communities, cores and peripheries, non-interacting blocks, etc. Developing methods to identify these patterns has been one of the main focus of Network Science; as a result, there are now a multitude of algorithms that tackle this task both efficiently and accurately.

While the definition of patterns—e.g., what is a community?—varies from one method to the other, it has been observed that different algorithms often yield the same mesoscopic description for any given network [1]. Furthermore, it has been shown that, in some cases, the similarities stem from the fact that a number of algorithms are operationally identical, despite differences in their formulation and conceptual frameworks [2,4].

To investigate these similarities more thoroughly, we present an all-encompassing formalism that draws on the theory of spin glasses [2,3]. In this formalism, objective functions are expressed as Hamiltonians, and mesoscopic patterns are identified by finding the ground state(s) of the system. We define a general Hamiltonian, and then show how most algorithms—e.g., modularity optimization and likelihood maximization of the stochastic block model—can be expressed as special cases of this Hamiltonian. In so doing, we delineate equivalence classes; we show that many algorithms are not only operationally but also mathematically identical. This generalizes many equivalence results, old [2] and recent [4].

[1] M. T. Schaub et al., arXiv:1611.07769 (2016)
[2] J. Reichardt, S. Bornholdt, Phys. Rev. E **74**, 016110 (2006)
[3] A. Decelle et al., Phys. Rev. Lett. **107**, 065701 (2011)
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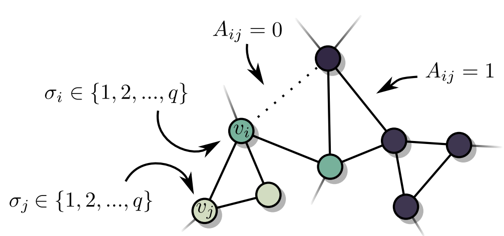

General spin glass

$$H = -\sum_{i,j} f_{ij}(A_{ij}, \sigma_i, \sigma_j)$$

Antiferromagnet

$$H = -\sum_{i,j}[A_{ij} g_{ij}(\sigma_i, \sigma_j) + (1 - A_{ij}) h_{ij}(\sigma_i, \sigma_j)]$$

(Equivalent to: Stochastic block model)

Isospin

$$H = -\sum_{ij} b_{ij}[A_{ij} + p_{ij}]\delta_{\sigma_i, \sigma_j} + C$$

(Equivalent to: Modularity, symmetric SBM, balanced-cut)

Figure 1: (*left*) General spin glass on a fixed network structure of adjacency matrix $\mathbf{A}$. The spin of site v_i, denoted σ_i, can take on q different orientations (here denoted by the color of nodes). Spins have pairwise couplings of strength $f_{ij}(A_{ij}, \sigma_i, \sigma_j) \in \mathbb{R}$. (*right*) Some special cases of the Hamiltonian, with equivalent objective functions for each case.