Functional resilience in dynamical complex networks with adaptive connectivity

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The brain is a notorious resilient system. After minor strokes, parts of the brain reorganize their structural connectivity and essentially recover their original functions. Although some dynamical effects of brain network failures on their activity have been found [1], most studies about resilient complex systems have so far focused on purely topological properties. This is due in part to the inherent high-dimensionality of connectomes with dynamical nodes. However, recent progresses suggest that the resilience analysis of many complex dynamical systems can be dramatically simplified by dimension reductions resulting from mean-field approximations [2,3]. We extend these previous works to study models of evolving networks in which nodes and edges weights are dynamical variables.

In our framework, the dynamics of a network with N nodes are described by $N(N+1)$ nonlinear coupled ODEs that govern the fast evolution of the neural activity (e.g., firing-rates) as well as the slow adaptation of the connectivity weights (e.g., Hebbian potentiation with saturation). Two global variables, the effective activity x_{eff} and the effective weighted connectivity β_{eff}, are used for predicting the global evolution of the whole system. When the adaptive connectivity is neglected, the resilience analysis can be easily done with bifurcation diagrams as in Fig. 1A. We prove, both numerically and theoretically, that x_{eff} captures more accurately the behavior of the network than the usual mean network activity. Structural perturbations, such as weak or strong attacks that respectively change weights or break edges, result in a modification of β_{eff}. If the latter reaches some critical value, β_c, the system undergoes a sudden transition and loses its resilience (Fig. 1B). The addition of adaptive connectivity leads to the emergence of new resilience patterns and often facilitates the recovery of the original network activity (Fig. 1C).

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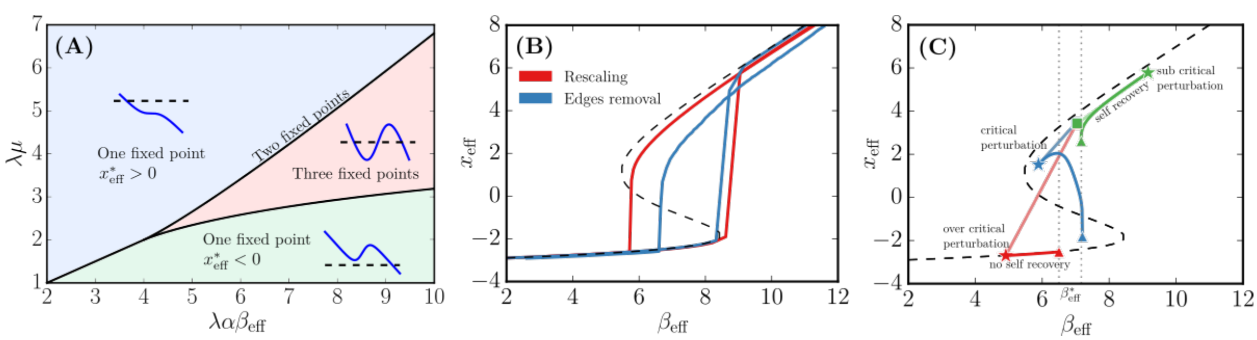

Figure 1: (A) Typical bifurcation diagram for the effective model without adaptive connectivity, where β, λ, α are dynamical parameters regulating the node dynamics while β_{eff} is effective weighted connectivity. (B) Global effective activity at equilibrium after weak (red line) or strong (blue line) attacks on static connections compared to the theoretical hysteresis curve (dashed line) obtained from mean-field theory. (C) Same as (B) but with adaptive connectivity. The square, stars, and triangles respectively denote the equilibria before an attack, just after an attack but before adaption, and after adaptation. Green line: resilience enabled by adaptation. The numerical solutions in (B) and (C) were produced from small random networks with 200 nodes and connectivity density $p = 0.2$.