Predicting synchronization regimes with dimension reduction on modular graphs

Vincent Thibeault[1,2], Guillaume St-Onge[1,2], Xavier Roy-Pomerleau[1,2] and Patrick Desrosiers[1,2,3].

1. Département de physique, de génie physique et d'optique, Université Laval, Québec (QC), G1V 0A6, Canada
2. Centre interdisciplinaire de modélisation mathématique de l'Université Laval, Québec (QC), G1V 0A6, Canada
3. Centre de recherche CERVO, Québec (QC), G1J 2G3, Canada

We study the synchronization of oscillator networks with community structures, such as two-star graphs and networks generated by the stochastic block model (SBM). We map high-dimensional (complete) dynamics unto low-dimensional (reduced) dynamics that captures the mesoscale features. This is achieved by introducing a spectral method for dimension reduction that generalizes and improves the method recently proposed in [1]. For the Kuramoto model on realizations of the SBM, the reduced dynamics describes the synchronized states correctly and, as a bonus, reveals the detectability limit for the SBM [2] [see FIG. (a)]. For the Sakaguchi-Kuramoto (SK) model on the mean adjacency matrix of the SBM, we prove that the spectral method leads to the same reduced model as the one obtained with the Ott-Antonsen Ansatz [3], highlighting the consistency of the approach with previous works. Moreover, we find new regions in the structural parameter space admitting chimeras, dynamical states characterized by the cohabitation of full synchronization in one community and partial synchronization in others. For the SBM, the size of chimera regions in the parameter space reaches a maximum when the communities are slightly asymmetric in size [FIG. (b) (Top)]. However, for the two-star graphs, the size of the chimera regions has a more complicated behavior and exhibits multiple step transitions with respect to the ratio of the periphery sizes [FIG. (b) (Bottom)].

[1] E. Laurence, N. Doyon, L.J. Dubé and P. Desrosiers, arXiv:1809.08285 (2018).

[2] J.-G. Young, P. Desrosiers, L. Hébert-Dufresne, E. Laurence and L.J. Dubé, *Phys.Rev.E*, **95**, p.062304 (2017).

[3] T. Kotwal, X. Jiang, and D.M. Abrams, *Phys.Rev.Lett.*, **119**, p.264101 (2017).

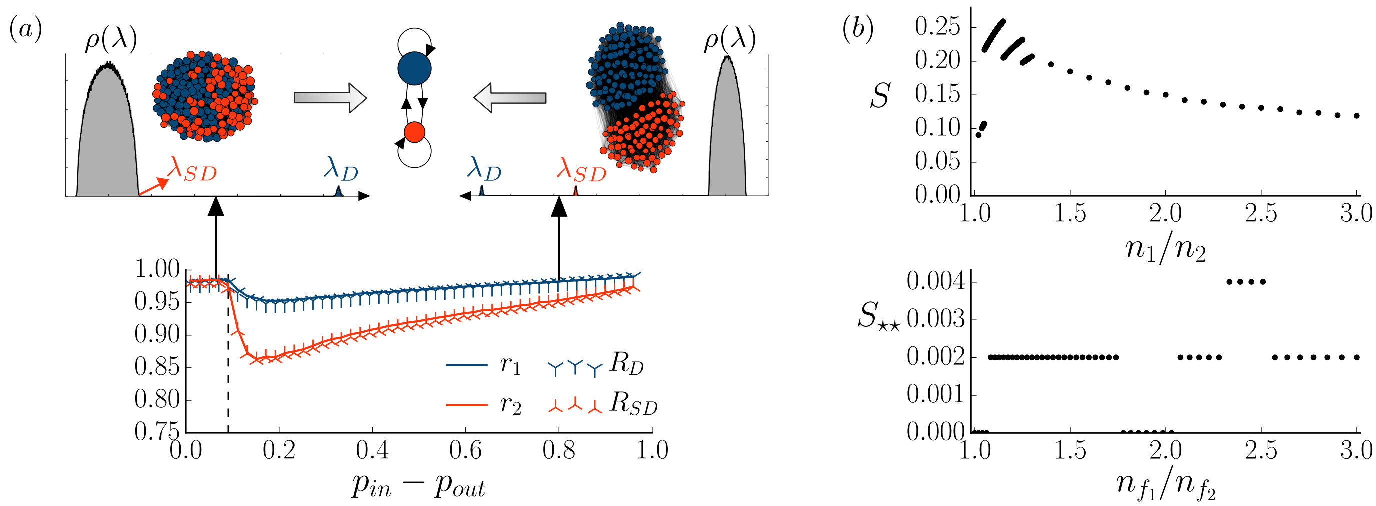

FIG. (a) (Top) Illustration of the spectral method for dimension reduction. Two realizations of the SBM with 200 nodes are given together with their spectral densities $\rho(\lambda)$. To get the reduced dynamics, we use the dominant eigenvalue λ_D and subdominant eigenvalue λ_{SD}, as well as their respective eigenvectors. The spectral method is accurate when communities are undetectable (left) and detectable (right). (Bottom) Synchronization parameters at equilibrium for each community for the complete dynamics, r_1 and r_2, compared with the corresponding parameters for the reduced dynamics, R_D and R_{SD}. The structural parameters p_{in} and p_{out} respectively describe the probability to be connected inside and outside each community. The dashed line indicates the detectability limit. FIG. (b) (Top) Size S of the chimera region for the SK dynamics (phase lag $\alpha = 1.45$) on the mean SBM with $n_1 + n_2 = 200$ nodes against the ratio n_1/n_2 of the community sizes. (Bottom) Size $S_{\star\star}$ of the chimera region for the SK dynamics (phase lag $\alpha = 0.94$) on two-star graphs with $n_{f_1} + n_{f_2} + 2 = 200$ nodes, where n_{f_μ} stands for the number of nodes in the periphery of the star μ, against the ratio n_{f_1}/n_{f_2}.