Persistent activity of neural dynamics on hierarchical networks

Dynamica research group

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Summary

Hierarchy has been hypothesized to facilitate the emergence of persistent activity [1]. We explore this statement with different hierarchical organizations on a binary neural dynamics. The multiple stable levels of activity, called **persistent activity**, are studied analytically and numerically.

- We analytically bound the number of intermediate states of activity for random, modular and hierarchical directed structures.
- **Hierarchical structures reduce the diversity of activities compared to modular structures.**
- For HPA structures, persistent activity emerges in a narrow window of parameters and shows delayed activation.

Model

Binary dynamics with spontaneous activation

Consider a graph composed of N neurons of binary activity $X_j(t)$ at time t and input activity $m_j(t) = \sum_i w_{ji} X_i(t)$.

Rate of activation

$$F(m) = \begin{cases} 1+\lambda & m \geq \mu \\ \lambda & \text{otherwise} \end{cases}$$

Rate of inactivation

$$R(m) = \nu$$

Master equation

$$\mathbb{E}\Big\{X_j(t+\Delta t)\Big\} = \mathbb{E}\Big\{[1 - X_j(t)]F(m_j(t))\Delta t + X_j(t)[1 - R(m_j(t))\Delta t]\Big\}$$

The master equation for the probability of being active is

$$P_j(t+\Delta t) = [1 - P_j(t)]F(m_j(t))\Delta t + P_j(t)[1 - R(m_j(t))\Delta t]$$

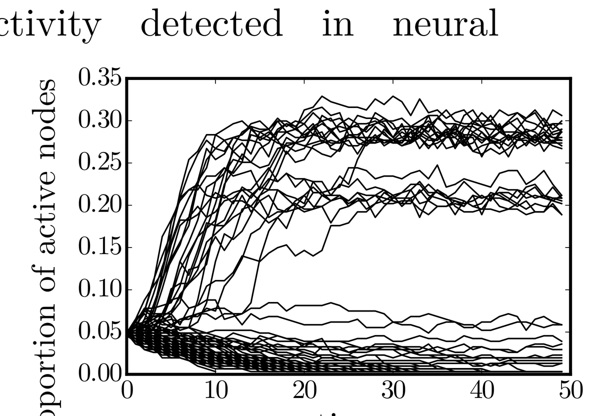
- Each node has at least one and at most two stable steady-states.
- Spontaneous activation enables a minimal level of activity.

Persistent activity

Persistent activity has been introduced by Kaiser et al. [1], to describe intermediate states of activity detected in neural networks.

- **Persistent activity is an intermediate and stable** state of activity.

In modular networks, we have found that persistent activity emerges from a weak communication between structures.



Example of persistent activity where each line is a single temporal evolution of the proportion of active nodes.

Bounds on the activity ($\lambda = 0$)

Random network

Let us consider a random network of density p. To be active, a node must have a number of active neighbors larger than a certain threshold parameter μ. A mean-field analysis provides two constraints on the number of active nodes r.

Inactive nodes

$$\frac{pr}{1+\nu} < \mu$$

Active nodes

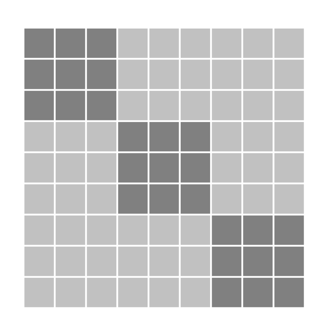
$$\frac{p(r-1)}{1+\nu} \geq \mu$$

Leading to

$$\frac{\mu(1+\nu)}{p} + p \leq r < \frac{\mu(1+\nu)}{p}$$

- **Two stable solutions exist: nodes are either all active or all inactive.**

Modular network



Adjancy matrix of a planted partition.

Let r be the number of active nodes from a planted partition of densities p_{in} and p_{out} and communities of size n.

Inactive nodes

$$r\frac{p_{\text{out}}}{1+\nu} < \mu$$

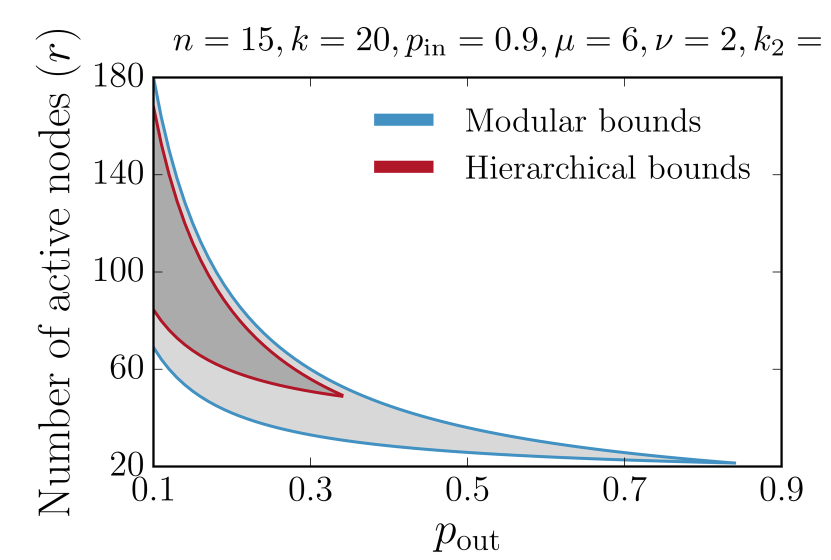
Active nodes

$$\frac{(n-1)p_{\text{in}}}{1+\nu} + \frac{p_{\text{out}}}{1+\nu}(r-n) \geq \mu$$

It leads to boundaries on the number of active nodes.

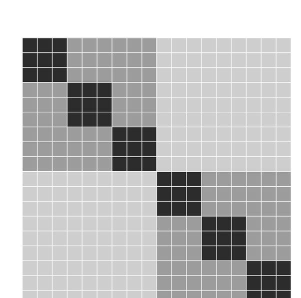
$$n + \frac{\mu(1+\nu)}{p_{\text{out}}} - \frac{(n-1)p_{\text{in}}}{p_{\text{out}}} \leq r < \frac{\mu(1+\nu)}{p_{\text{out}}}$$

- The number of active nodes is highly constrained by the dynamics and the densities of the network.
- **Persistent activity of modular networks is bounded.**



Example of the bounds for the number of active nodes for modular and hierarchical structures. The gray zones are the bounded regions that contain the admissible values of r for a given p_{out}. The parameters p_1 and p_2 are set to obtain the same number of edges and nodes.

Hierarchical network



Adjacency matrix of an hierarchical planted partition.

Let us consider a structure of 2 hierarchical levels of organization and k_2 structures at the highest level and k_1 communities of size n at the lowest level. Densities between structures are p_{in}, p_1 and p_2 (see figure on the left). We apply a similar treatment as the modular structure to obtain constraints on the number of active nodes.

$$[\mu(1+\nu) + p_1 n - p_{\text{in}}(n-1)]\frac{k_2}{(p_1 - p_2) + k_2 p_2} \leq r < \mu(1+\nu)\frac{k_2}{(p_1 - p_2) + k_2 p_2}$$

We have compared the hierarchical constraints to the modular constraints for networks of same size and the same number of edges.

- For equivalent networks, hierarhical structures have smaller regions of possible number of active nodes than modular structures.
- **Hierarchy reduces the diversity of persistent activities.**

HPA structures

Hierarchical preferential attachment (HPA)

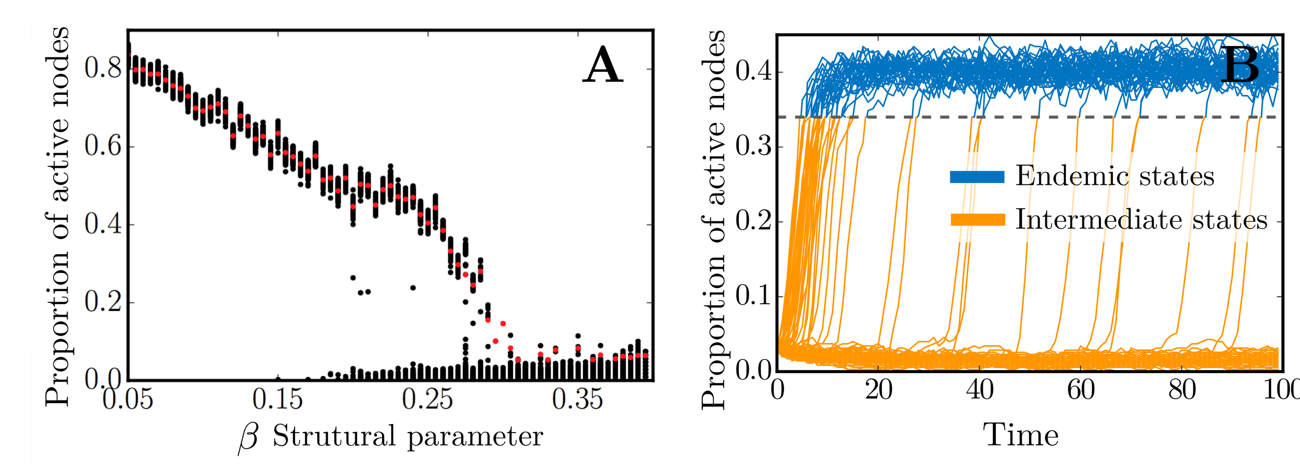
HPA has been introduced in Ref. [2] and is caracterised by

- Scale free degree distribution.
- Scale free community size and membership distributions.
- Realistic model for hierarchical structures.

Observation of persistent activity

We have applied the binary dynamics on HPA networks of four hierarchical levels. We have controlled the structure using a parameter β.

- **Intermediate states of activity emerge close to the critical threshold of activation.**
- **The transition to endemic state is delayed.**



A: Single active fraction (black dots) and average active fraction (red dots) of the network as a function of a structural parameter. The binary dynamics is simulated using $\mu = 3, \nu = 0.1, \lambda = 0.001$

B: Active fraction for a given structural parameter β=0.25 close to the threshold as function of time for 100 simulations.

Bibliography

[1] Kaiser M., Görner M., and Hilgetag C.C., Criticality of spreading dynamics in hierarchical cluster networks without inhibition, New J. Phys., **9** (2007), p. 110.

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