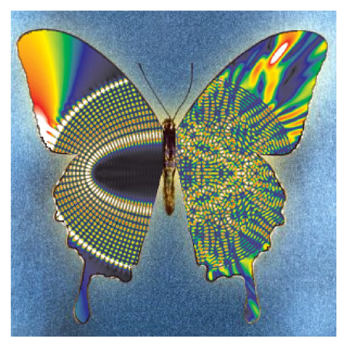
Numerical design and optimization strategies for annular silica microcavities



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1. Experimental setup

High Q-factor erbium-doped silica **annular cavities** are ideal candidates for **integrated directional lasers**. Non-uniform emission can be achieved without significantly spoiling the Q-factor of the unperturbed cavity (no inclusion).

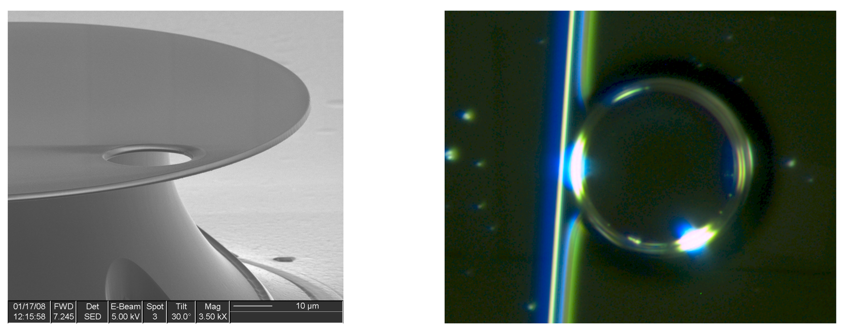


Fig. 1: Scanning electron microscopy (left) and optical characterization setup (right) of a silica annular microcavity.

Fabrication process: The silica (SiO_2, refractive index ~ 1.5) disk is patterned using standard **photolithographic methods** followed by wet HF etching. A dry underetching of the silicon base releases the outer region of the disk. A CO_2 laser reflow is performed to generate a **toroidal cavity**. Microstructures (e.g. holes) can be engraved using a focused ion beam (Fig. 1).

Optical characterization: Light is coupled inside the cavity using a 2 µm **tapered fiber** (Fig. 1). The resonant spectrum is obtained using a tunable laser source. Once tuned to a resonant frequency, the far-field emission may be recorded using various methods. A reflecting stage coupled to an infrared camera has been developed for this purpose.

Goals

Acquire **design rules** to harness the far-field characteristics of integrated optics devices. To guide the experiment, we have pursued **numerical simulations** of the associated dynamics. Two types of **dynamics** are studied concomitantly.

1. The wave dynamics (boundary element method)
2. The classical dynamics (ray-escape simulations)

2. Boundary element method for annular cavities

The **wave dynamics** of the annular cavity is described by the 2D Helmholtz equation

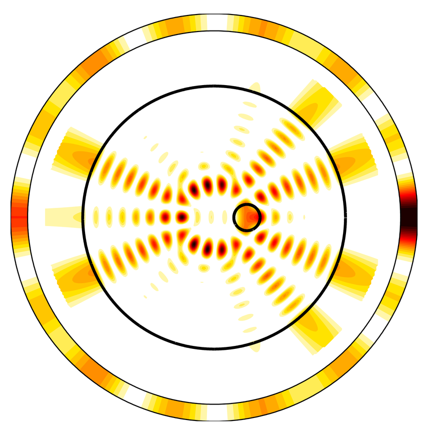
$$\left[\nabla^2 + n^2(\boldsymbol{r})k^2\right]\psi(\boldsymbol{r}) = 0.$$

Using Green's second identity, one obtains the following boundary integral equation

$$\psi(\boldsymbol{r}') = \oint_{\Gamma_j} d\boldsymbol{s}\cdot\left[\psi(\boldsymbol{r})\nabla G(\boldsymbol{r},\boldsymbol{r}';k) - G(\boldsymbol{r},\boldsymbol{r}';k)\nabla\psi(\boldsymbol{r})\right]$$

where $G(\boldsymbol{r},\boldsymbol{r}';k)$ is the homogeneous medium **Green's function**. We have implemented a generalization of the ***boundary element method*** [1]. This generalization allows to find resonant modes of annular cavities (more generally, to solve the problem in **multiply connected domains**). The numerical procedure can be summarized as follows

1. A boundary integral equation is obtained for the three dielectric domains (inclusion, disk, surroundings) shown on Fig. 2
2. The boundary is discretized in order to transform integral equations in a non-linear eigenvalue problem
3. The complex resonant modes are found using **analytic** (unperturbed) solutions as starting points for the eigenvalue search [2]. Every analytic solution is labeled by an angular quantum number m and possesses a resonant wavenumber $k_{\text{res}} = k' - ik''$.



Holey structures?
The BEM algorithm for annular cavities also applies to holey fibers, photonic crystal cavities and fibers. In all cases, the axial field component is uncoupled

3. Ray-escape simulations

The annular cavity consists of a circular disk of radius R and refractive index n_2, surrounded by a medium of index n_1, with a circular inclusion of radius R_a and index n_1 displaced a distance d from the cavity center.

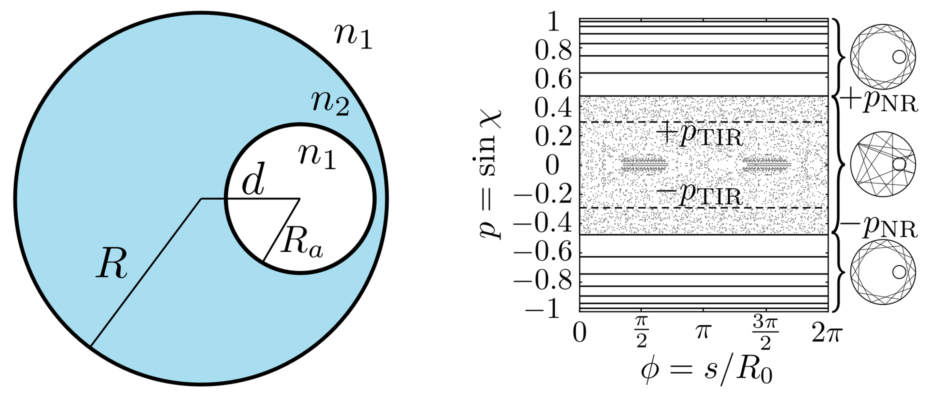


Fig. 2: Annular cavity geometry and associated phase space

The **annular billiard** provides a useful model for the emission properties of annular cavities. Three important phase-space limits exist in the case of the open billiard

1. The non-regular (NR) boundary separating regular and chaotic trajectories, $p_{\text{NR}} = (d + R_a)/R$
2. The total internal reflection (TIR) limit, $p_{\text{TIR}} = n_1/n_2$
3. The semiclassical wave momentum, $p_m = m/n_2k'R$.

Ray-escape simulations give us access to **universal** far-field characteristics, those expected in the semi-classical regime.

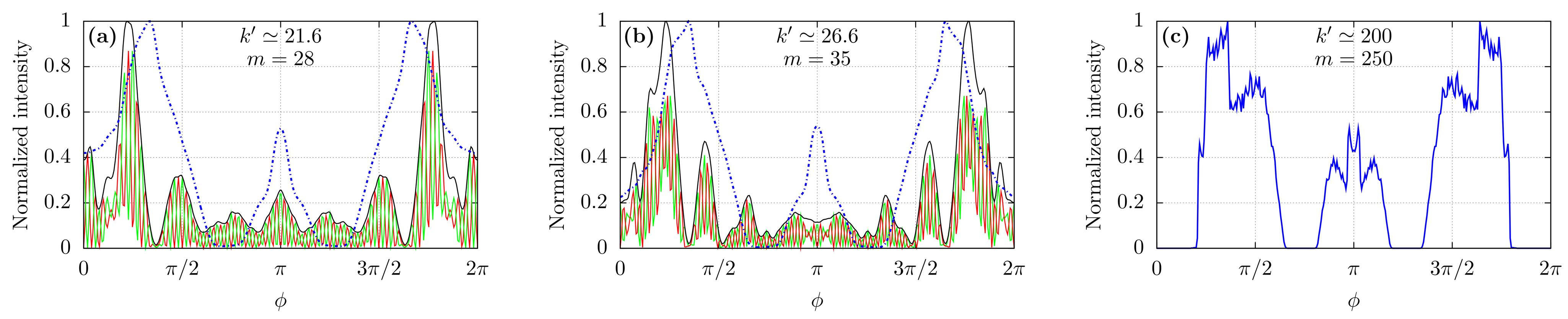
Choice of geometric parameters

An eccentric inclusion ($d \neq 0$) and an appropriate choice of R_a can induce non-uniform emission while preserving the Q-factor of WGMs [3]. Two conditions must be satisfied

1. TIR region embedded in NR region ($p_{\text{TIR}} < p_{\text{NR}}$)
2. WGM localized outside NR region ($p_m > p_{\text{NR}}$).

Parameter	d/R	R_a/R	p_{NR}	p_{TIR}
Value	0.5	0.2	0.7	0.67

4. Ray-wave correspondence and universality of far-field emission



Comparison of far-field intensities computed via the boundary element method and ray-escape simulations (**blue**). Odd (**red**) and even (**green**) symmetry modes of the inhomogeneous annular cavity are **quasi-degenerate**, and the far-field intensity is therefore computed as $I = |\psi_o|^2 + |\psi_e|^2$ (**black**). **(a)** Resonant mode with unperturbed $Q \simeq 10^4$. **(b)** Resonant mode with unperturbed $Q \simeq 10^5$. **(c)** A case closer to experimental reality (infrared wavelength, $R = 50$ µm) which is at present beyond the limit of our wave simulations.

The ray-escape and wave results agree quite well with respect to the dominant **emission directions**, even for relatively low wavenumbers. The global shape for a given configuration displays **universal** behaviour regardless of the wavenumber regime as long as $p_m \gg p_{\text{NR}}$.

5. How to improve collimation?

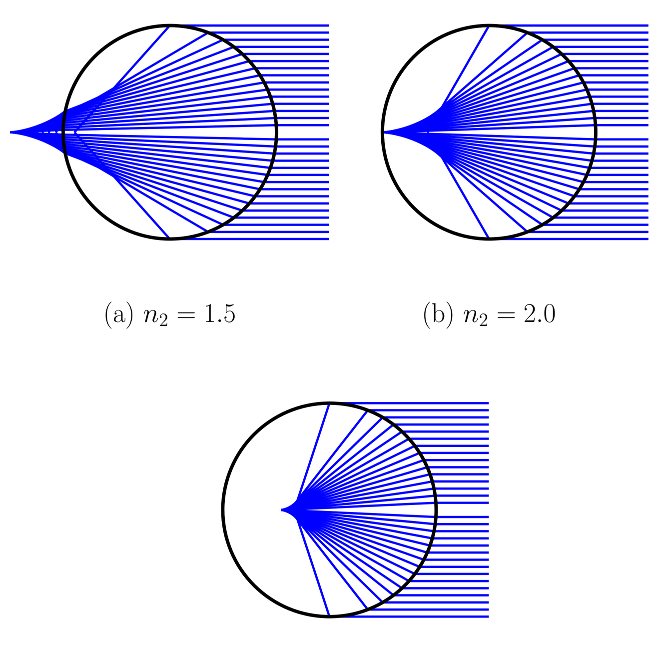
Ray-escape and wave results suggest that the triple-peaked far-field profile observed for silica cavities is similar for an inner inclusion, **regardless of the geometry**.

A simple model based on **paraxial ray-optics** can help improve collimation. Circular cavities (the backbone of annular cavities) are approximated as paraxial lenses and **focal points** are found. A **point source** placed at a focal point will result in highly collimated emission. For a silica cavity, this point is located **outside the boundary**.

Promising designs

While this model is mostly accurate in the regime $R_a \ll \lambda$ ("point" inclusion), it suggests many interesting designs that might exhibit better emission directionality

- **Fig. 3a**: Replacing an inclusion by an **exclusion** (coupled silica cavities, photonic molecules)
- **Fig. 3b**: Cavities with a defect **on the boundary**, e.g. [4]
- **Fig. 3c**: **Silicon-On-Insulator** (SOI) annular cavities.



(a) $n_2 = 1.5$ (b) $n_2 = 2.0$ (c) $n_2 = 3.2$

Fig. 3: Focal points of a circular cavity

6. Conclusions and outlooks

The ray-wave correspondence allowed us to infer **universal** far-field properties of silica annular cavities. This permits computation beyond the limit of wave simulations.

A simple geometric model was developed in order to help improve **emission directionality** via new designs (modified **geometries** as well as different **refractive indices**).

The next experimental step is to obtain reliable **far-field measures**, while a numerical investigation of promising geometries/medium combinations is being made.

References

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