Functional resilience in neural networks

Mediterranean School of Complex Networks

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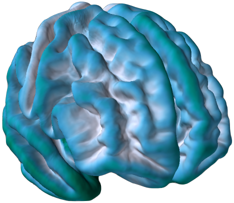
Resilience
Ability to recover the original state in a reasonnable short period of time.

Robustness - *Opposite of vulnerability*
Difficulty to modify the state of a system.



The brain is resilient

Plasticity, compensation, ...

The details and strategies are still unknown

Nodes

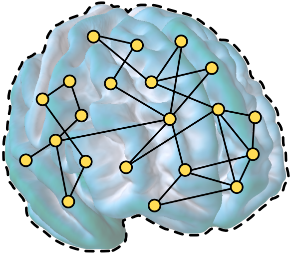
Neurons of activity $x_i(t)$

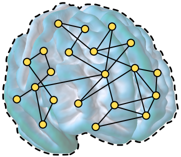
Edges

Synapses of weight $w_{ij}(t)$

Perturbations

$\Delta x_i(t)$, $\Delta w_{ij}(t)$, edges/nodes removal, ...



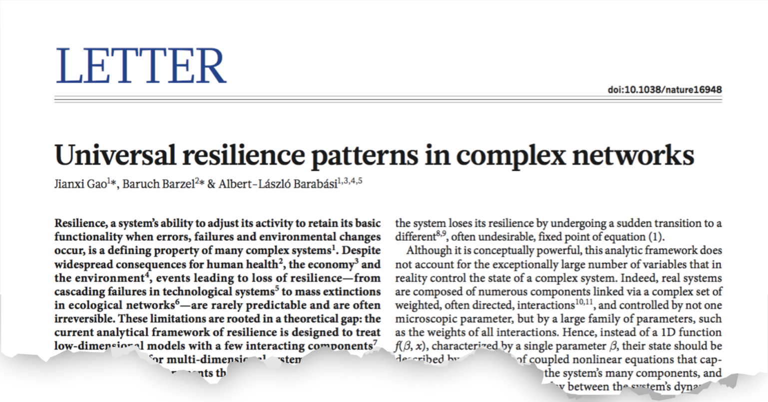


Effective formalism

- Description
- Application to neural networks
- Approximations and errors

Adaptive connectivity

- Special behaviors
- Measures of resilience


LETTER

doi:10.1038/nature16948

Universal resilience patterns in complex networks

Jianxi Gao[1]*, Baruch Barzel[2]* & Albert-László Barabási[1,3,4,5]

Resilience, a system's ability to adjust its activity to retain its basic functionality when errors, failures and environmental changes occur, is a defining property of many complex systems[1]. Despite widespread consequences for human health[2], the economy[3] and the environment[4], events leading to loss of resilience—from cascading failures in technological systems[5] to mass extinctions in ecological networks[6]—are rarely predictable and are often irreversible. These limitations are rooted in a theoretical gap: the current analytical framework of resilience is designed to treat low-dimensional models with a few interacting components[7]

the system loses its resilience by undergoing a sudden transition to a different[8,9], often undesirable, fixed point of equation (1).

Although it is conceptually powerful, this analytic framework does not account for the exceptionally large number of variables that in reality control the state of a complex system. Indeed, real systems are composed of numerous components linked via a complex set of weighted, often directed, interactions[10,11], and controlled by not one microscopic parameter, but by a large family of parameters, such as the weights of all interactions. Hence, instead of a 1D function $f(\beta, x)$, characterized by a single parameter β, their state should be

Effective formalism

Presented by Gao *et al.* 2016

N-dimensional complete system

$$\dot{x}_i = F(x_i) + \sum_{j=1}^{N} w_{ij} G(x_i, x_j)$$

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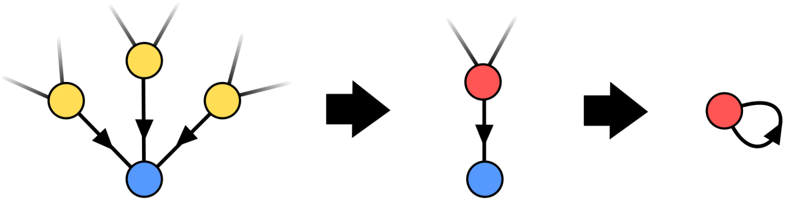

N-dimensional complete system

$$\dot{x}_i = F(x_i) + \sum_{j=1}^{N} w_{ij} G(x_i, x_j)$$

1-dimensional effective system

$$\dot{x}_{\text{eff}} = F(x_{\text{eff}}) + \beta_{\text{eff}} G(x_{\text{eff}}, x_{\text{eff}})$$

$$x_{\text{eff}} = \mathscr{L}(\boldsymbol{x}) = \frac{\sum_{ij} w_{ij} x_j}{\sum_{ij} w_{ij}} \quad ; \quad \beta_{\text{eff}} = \mathscr{L}(\boldsymbol{s}) = \frac{\sum_{ijk} w_{ij} w_{jk}}{\sum_{ij} w_{ij}}$$

$\mathscr{L}(\boldsymbol{x})$ = Neighborhood average of $\boldsymbol{x}$

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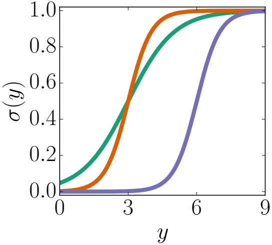
Neural dynamics - Hopfield model

$$\dot{x}_i = -x_i + \sum_j w_{ij} \sigma \left[\lambda \left(x_j - \mu \right) \right]$$

$$\sigma(y) = \frac{1}{1 + \mathrm{e}^{-y}}$$



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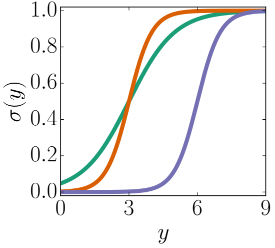

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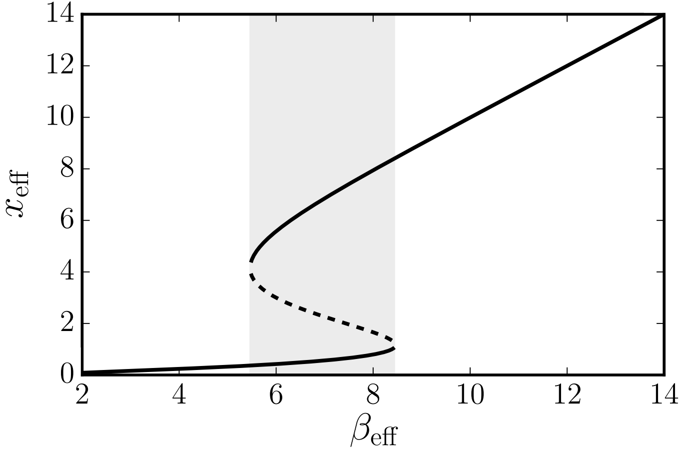
Stationnary state $\dot{x}_{\text{eff}} = 0$



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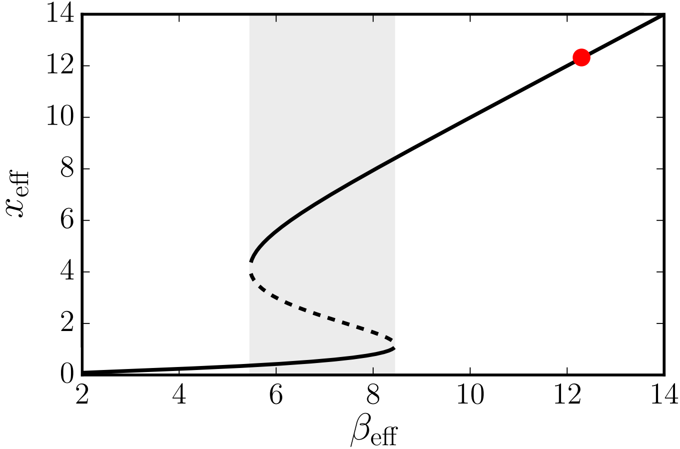

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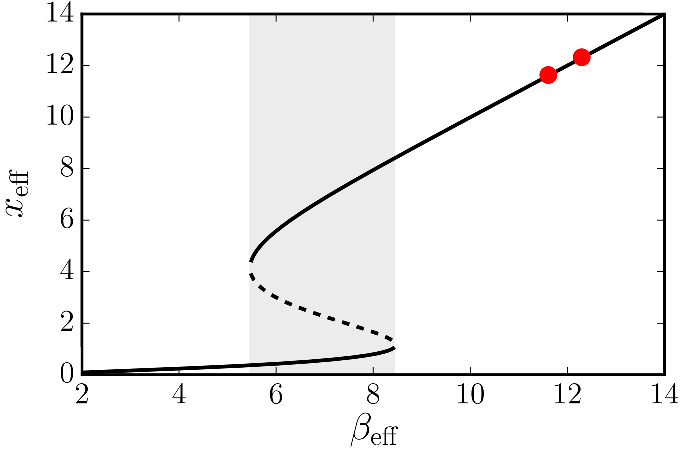

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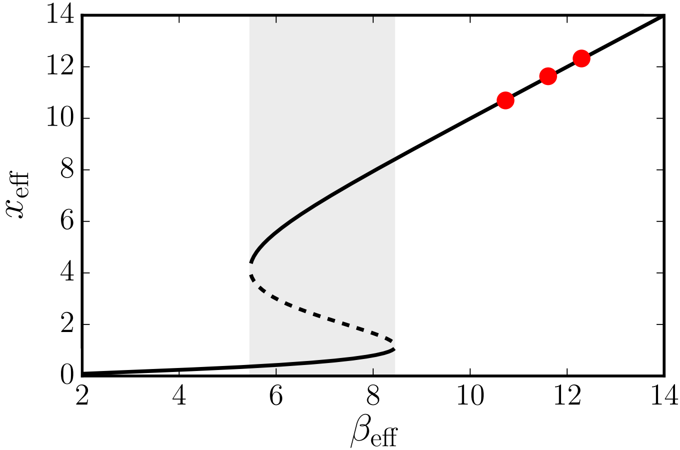

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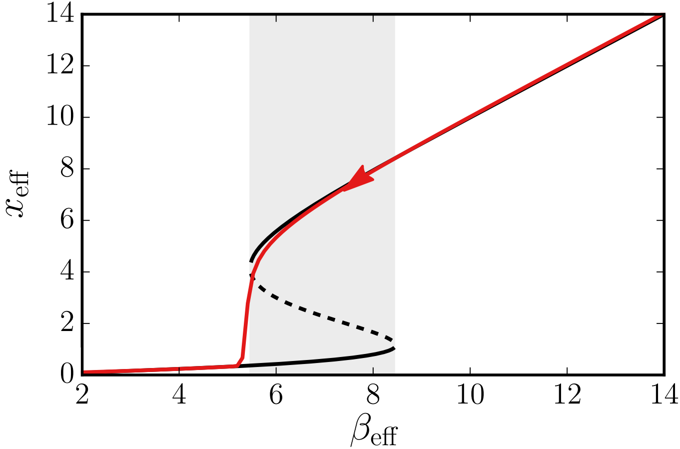

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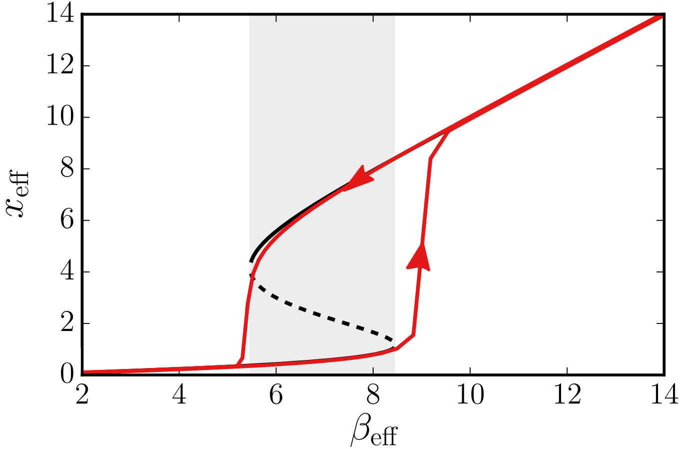

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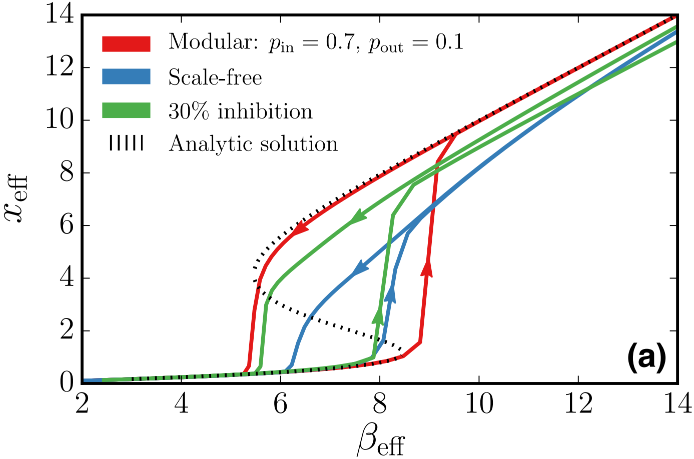

$$\dot{x}_{\text{eff}} = -x_{\text{eff}} + \beta_{\text{eff}} \sigma \left[\lambda \left(x_{\text{eff}} - \mu \right) \right]$$

Good approximation for

- Homogeneous network
- Low inhibition
- High reciprocity $w_{ij} = w_{ji}$

Adaptive connectivity

Resilience

Ability to **recover** the original state in a reasonnable short period of time.

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Modified **Hebb's rule**

$$\dot{w}_{ij} = \tau_S^{-1}(\sigma_i\sigma_j - w_{ij}\sigma_j^2) \qquad ; \qquad \sigma_i = \sigma[\lambda(x_i - \mu)]$$

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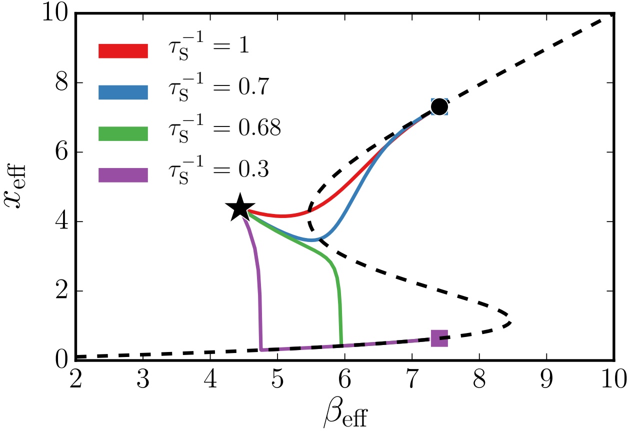


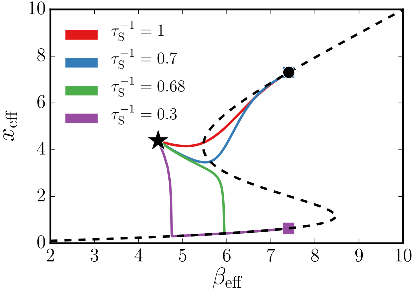
How to quantify resilience ?

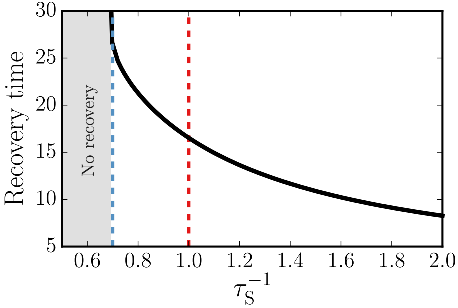
How to quantify resilience ?

- Recovery time
- Energy of recuperation
- Maximum damage
- Sensibility $\frac{dx_{\text{eff}}}{d\beta_{\text{eff}}}$

Recovery time

Time to return in the surroundings of the original state





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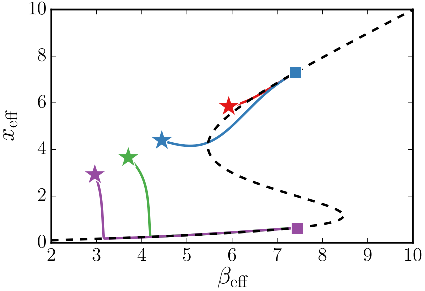
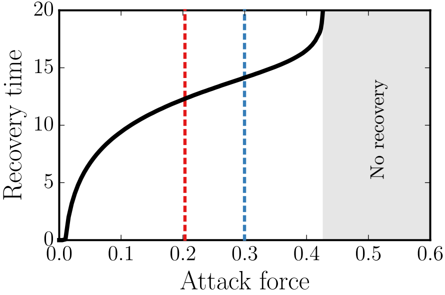


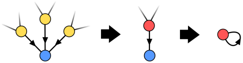


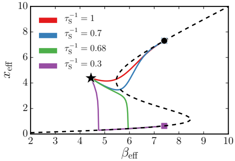
Critical slowing down

Conclusion

Effective formalism

- Simple to use on neural dynamics.
- Valid for homogeneous, low inhibition and high reciprocity.





Resilience

- Introduce adaptive connectivity
- Recovery time is a good indicator of catastrophe

Collaborators

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