ALIFE 2020

Localization, bistability and optimal seeding of contagions on higher-order networks

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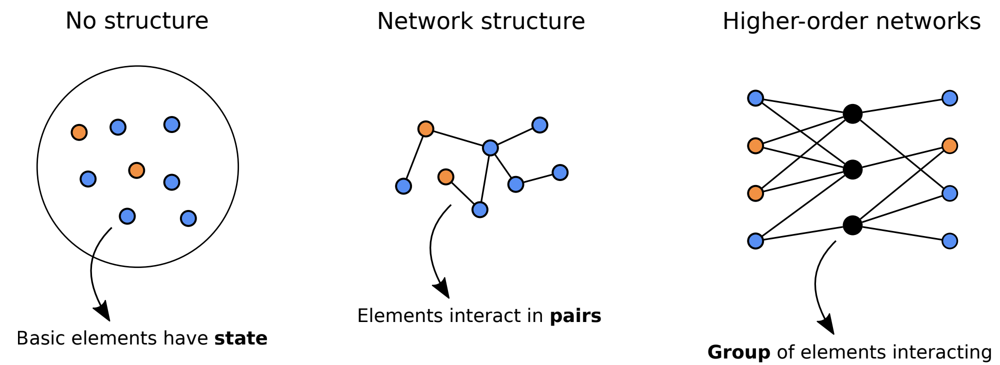
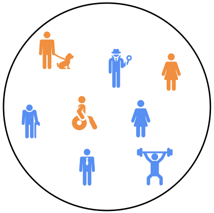
Representations of complex systems



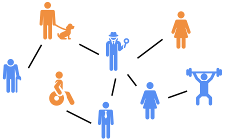
- **State** : neuronal activity, political allegiance, species abundance
- **Pair** interaction : synapse, friendship, predator-prey relationship
- **Group** (higher-order) interaction : workplace environment, ecosystem

Contagion dynamics

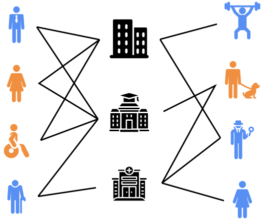
No structure



Network structure



Higher-order networks



Susceptible

Infected

*Icons made by Freepik, catkuro, Smashicons and Pixel perfect from "www.flaticon.com"

Outline

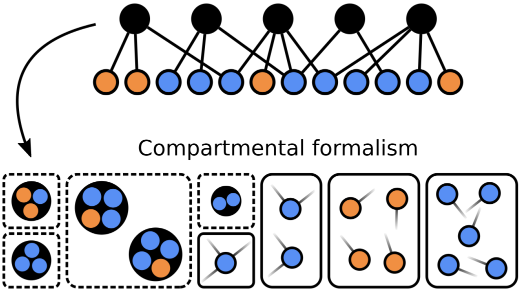
Goal of the presentation

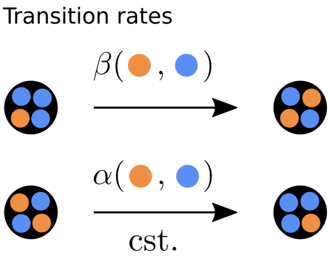
- Promote higher-order network (HON) representations of complex systems
- Introduce an accurate method to describe stochastic dynamics on HONs

Outline

1. Approximate master equations
2. Applications to contagion dynamics
    - Localization of epidemics
    - Bistability
    - Optimal seeding

Approximate master equations





Mean-field equations for nodes

$$\frac{\mathrm{d}s_m}{\mathrm{d}t} = 1 - s_m - m\, r\, s_m \ .$$

Approximate master equations for groups

$$\begin{aligned}\frac{\mathrm{d}f_{n,i}}{\mathrm{d}t} = &\ (i+1) f_{n,i+1} - i f_{n,i} \ , \\ &- (n-i)\Big[\beta(n,i) + \rho\Big] f_{n,i} \ , \\ &+ (n-i+1)\Big[\beta(n,i-1) + \rho\Big] f_{n,i-1} \ .\end{aligned}$$

Example

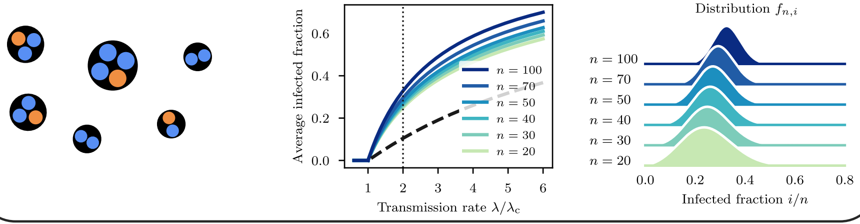
$f_{3,2}$

s_3

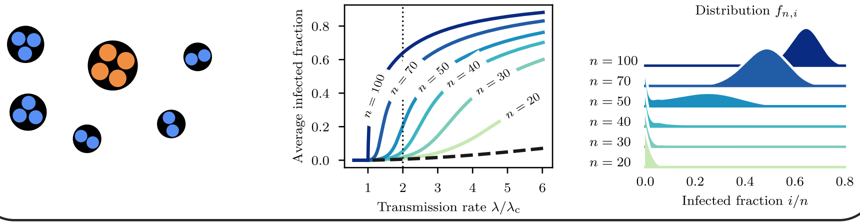
- $s_m(t)$: fraction of susceptible nodes with membership m
- $f_{n,i}(t)$: fraction of groups of size n with i infected
- $\beta(n,i)$: local infection rate
- $r(t)$, $\rho(t)$: mean-field couplings

SIS model : $\beta(n,i) = \lambda i$

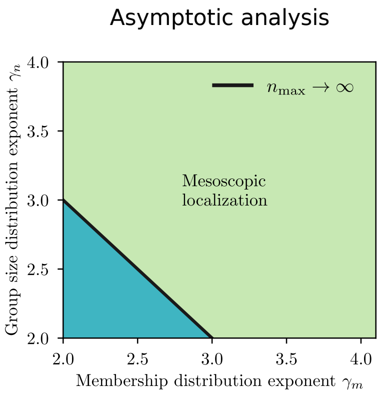
Delocalized regime

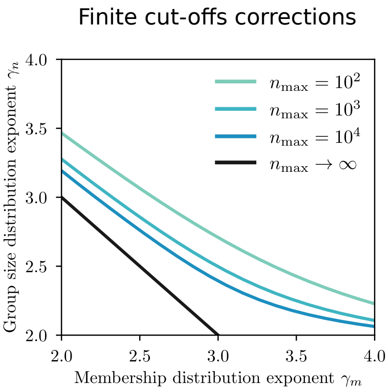


Mesoscopic localization regime



Localization regimes



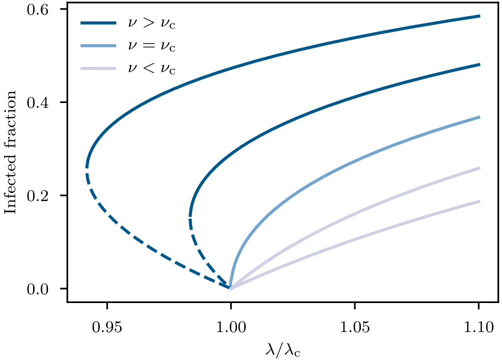


- Group size distribution : $p_n \sim n^{-\gamma_n}$ with cut-off $n_{\max}$
- Membership distribution : $g_m \sim m^{-\gamma_m}$ with cut-off $m_{\max} = n_{\max}$

Simple model of social contagion

$$\beta(n,i) = \lambda i^{\nu}$$

- $\nu < 1$: inhibition effect
- $\nu = 1$: SIS model
- $\nu > 1$: reinforcement effect



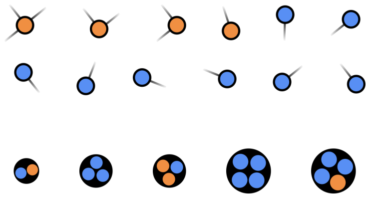
Goal : *Maximize* $\dot{I}(0)$ *by distributing wisely* $I(0) = \epsilon \ll 1$.

Rules

- We set $\lambda > \lambda_c$ so that $I^* = 0$ is unstable
- You can choose among two approaches
  1. *Influential spreaders* : engineer node set $\{s_m(0)\}$
  2. *Influential groups* : engineer group set $\{f_{n,i}(0)\}$
- The unchosen set is distributed randomly, i.e.

$$f_{n,i}(0) = \binom{n}{i} \epsilon^i (1-\epsilon)^{n-i} \quad \text{or} \quad s_m = 1 - \epsilon \; \forall m \; .$$

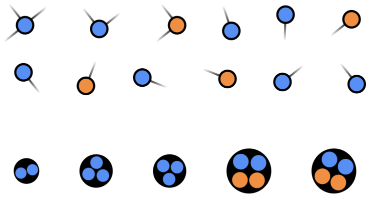
Influential spreaders



Optimal strategy

Infect nodes with highest available membership m

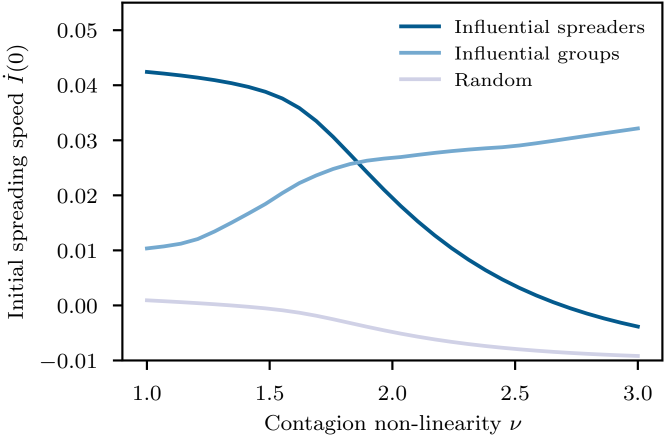
Influential groups



Optimal strategy

Favor most *profitable* group configurations (n, i) as measured from $R(n,i) = \beta(n,i)(n-i)/i$

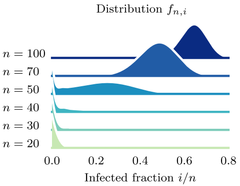
Influential groups beat influential spreaders in nonlinear contagions

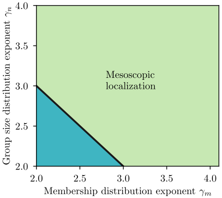


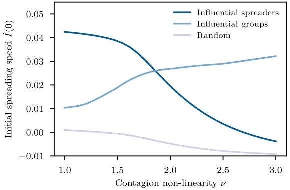
What can higher-order network representations do for you?

- New insights due to the focus on groups of elements
- Analytical results to guide further exploration
- The framework presented can be applied to various dynamical processes
  - Voter models, evolutionary game theory, etc.

$$\frac{\mathrm{d}f_{n,i}}{\mathrm{d}t} = (i+1)\left[\alpha(n,i+1) + \rho_1\right] f_{n,i+1} - i\left[\alpha(n,i) + \rho_1\right] f_{n,i} \,, \\ - (n-i)\left[\beta(n,i) + \rho_2\right] f_{n,i} + (n-i+1)\left[\beta(n,i-1) + \rho_2\right] f_{n,i-1} \,.$$







Epidemic localization

Vincent Thibeault, Antoine Allard, Louis J. Dubé, Laurent Hébert-Dufresne

Preprints : arXiv:2004.10203 and arXiv:2003.05924

Bistability and optimal seeding

Iacopo Iacopini, Giovanni Petri, Alain Barrat, Vito Latora, Laurent Hébert-Dufresne

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