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BURSTY EXPOSURE ON HIGHER-ORDER NETWORKS LEADS TO NONLINEAR INFECTION KERNELS

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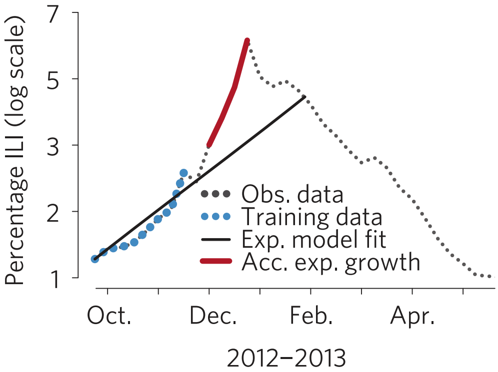
This is because we assume that the risk of infection is linear

$$\theta \propto I$$

Superexponential spread of Influenza-Like-Illness [1]



1. Scarpino, S. V., Allard, A., & Hébert-Dufresne, L. (2016). The effect of a prudent adaptive behaviour on disease transmission. Nature Physics, 12(11), 1042-1046.

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(i) Why assume linearity?

(ii) When is linearity valid?

(iii) What other forms could it take?

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Take-home message

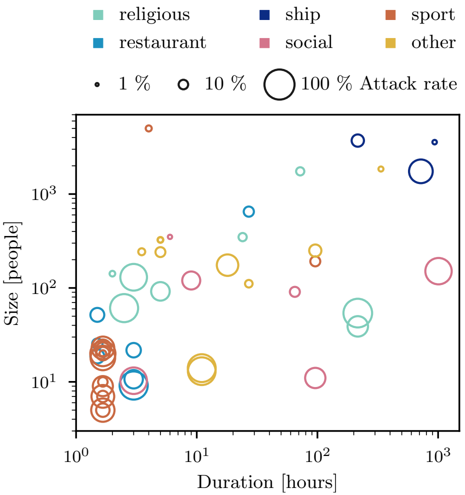
(iii) : Assuming bursty exposure to infection, we should consider

$$\theta \propto I^{\nu} \quad \text{with } \nu \in \mathbb{R}^{+} .$$

Model features

1. Explicit group interactions
2. Heterogeneous temporal patterns
   - Duration of events τ

$$P(\tau) \propto \tau^{-\alpha-1}$$



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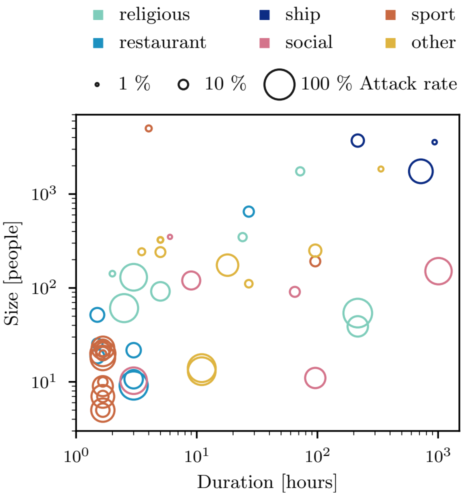
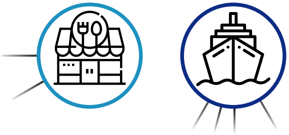
3. *Minimal infective dose*

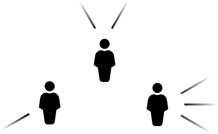


Feature # 1 : Explicit group interactions – bipartite structure

Environments

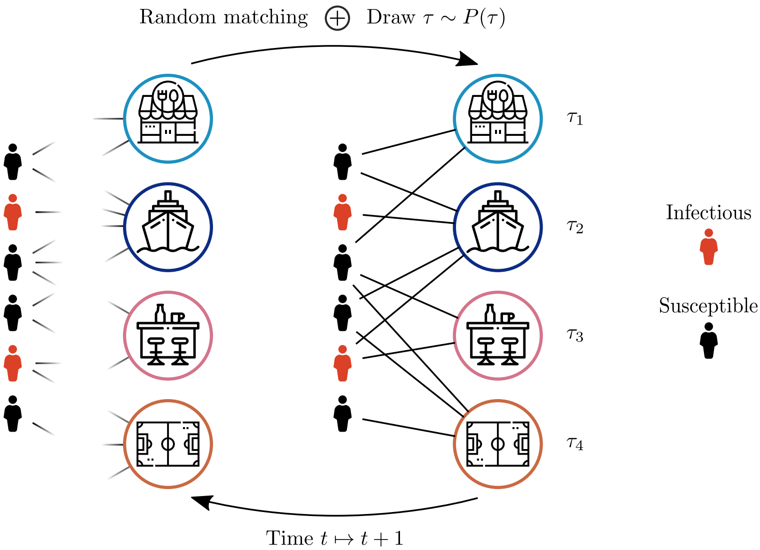


Individuals



Icons made by Freepik from www.flaticon.com

Feature # 2 : heterogeneous temporal patterns



Feature #3 : minimal infective dose

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Universal Behavior in a Generalized Model of Contagion

Peter Sheridan Dodds[1,*] and Duncan J. Watts[2,3,†]

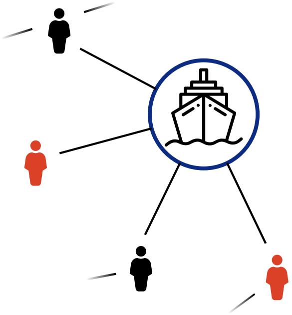
Infective dose model

- The fraction of infected individuals is ρ
- Individuals receive a dose $\kappa \sim \pi(\kappa; \rho, \tau)$
- The mean dose received is

$$\langle \kappa \rangle \propto \rho\tau$$

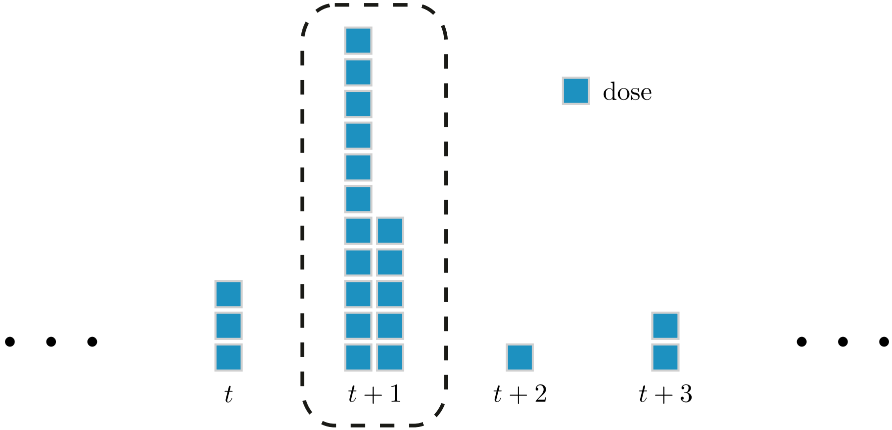
- An infection is triggered if $\kappa \geq K$, with probability

$$\bar{\Pi}(K; \rho, \tau) = \int_K^\infty \pi(\kappa; \rho, \tau) \mathrm{d}\kappa$$



Bursty exposure

Because of the heavy-tailed distribution $P(\tau) \propto \tau^{-\alpha-1}$



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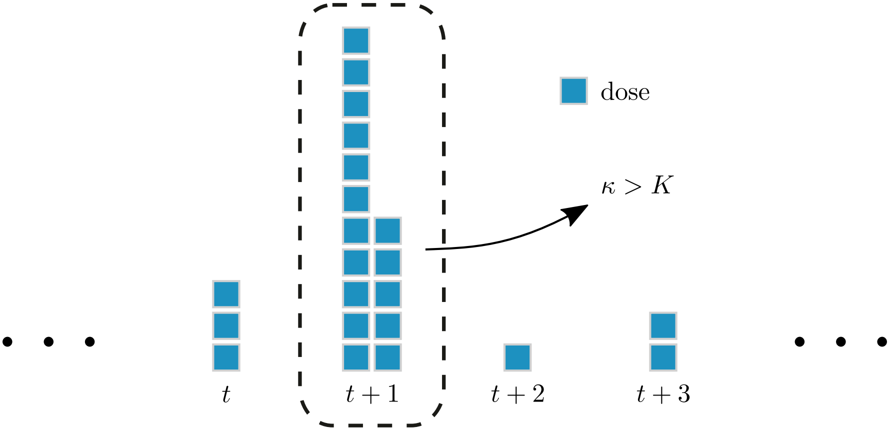

Emergent nonlinear infection kernel

The probability of getting infected in an environment

$$\theta(\rho) = \int P(\tau)\bar{\Pi}(K;\rho,\tau)\mathrm{d}\tau \ .$$

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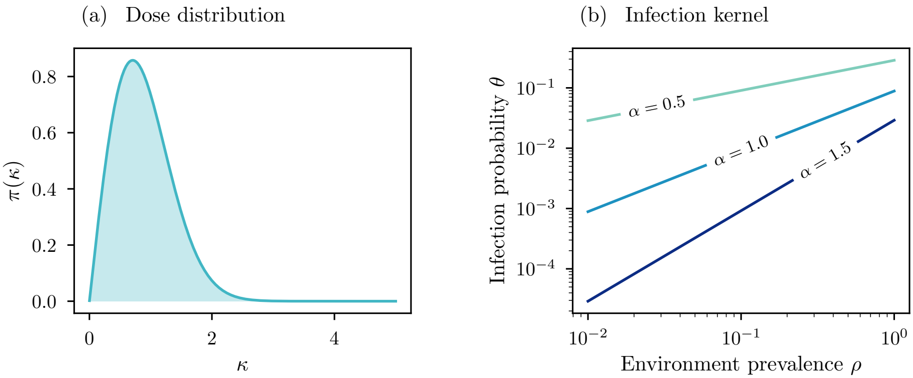
Assuming :

1. $P(\tau) \propto \tau^{-\alpha-1}$;
2. *Some technical conditions for the asymptotic analysis* ;

for a large class of dose distribution π, we recover the *universal* infection kernel

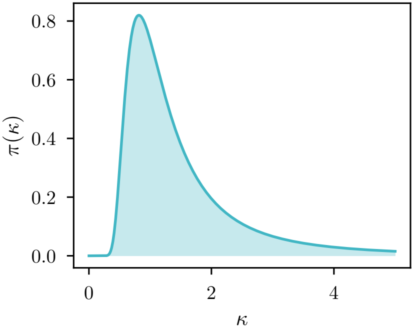
$$\boxed{\theta(\rho) \propto \rho^{\alpha}}$$

Weibull dose distribution

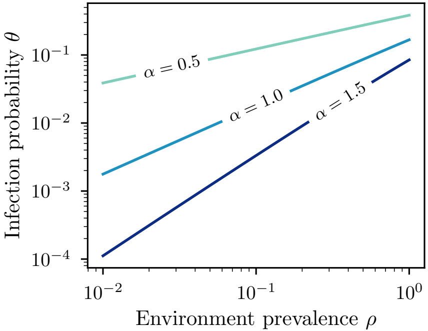


Frechet dose distribution

(a) Dose distribution

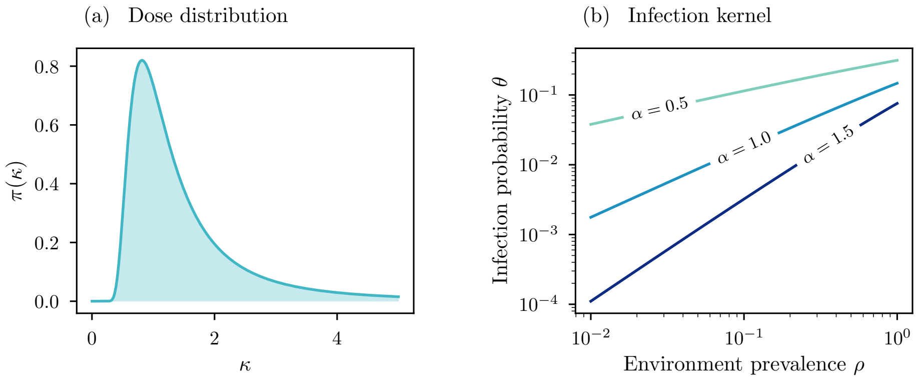


(b) Infection kernel

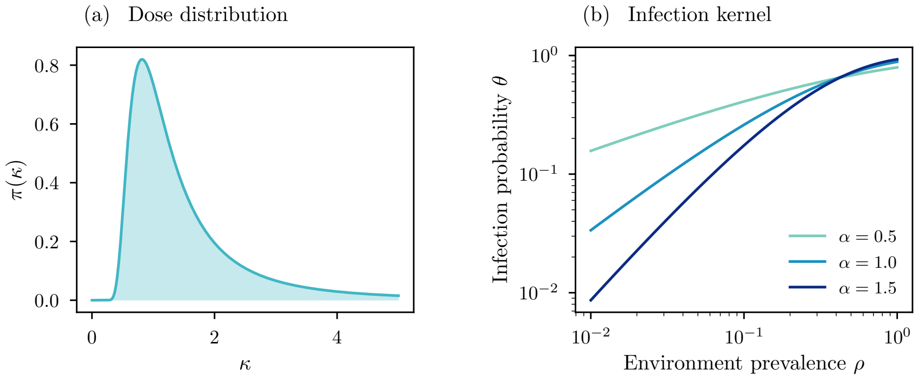


Asymptotically power-law duration of events distribution

$P(\tau) \propto \tau^{-\alpha-1}$ only for large τ



Conditions for the asymptotic analysis partially satisfied



When is linearity valid?

- $\alpha = 1 \quad (P(\tau) \propto \tau^{-\alpha-1})$
- π is a Poisson distribution and $K = 1$
- Some other limit cases

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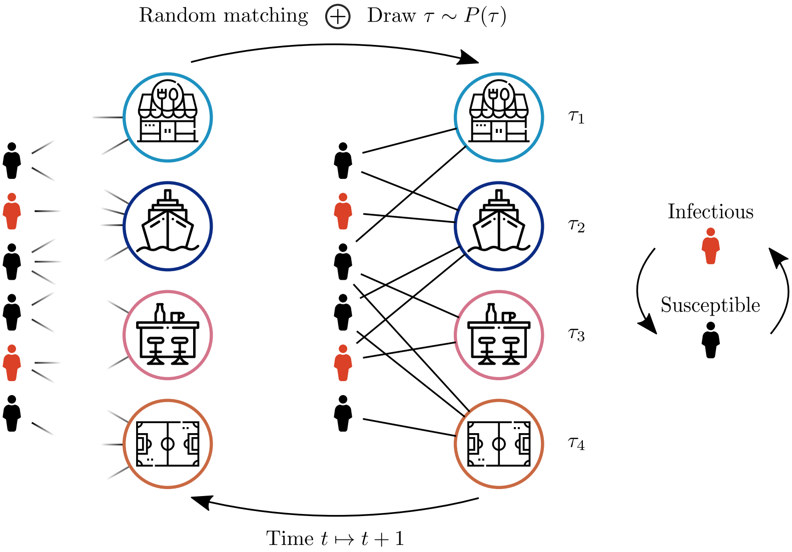
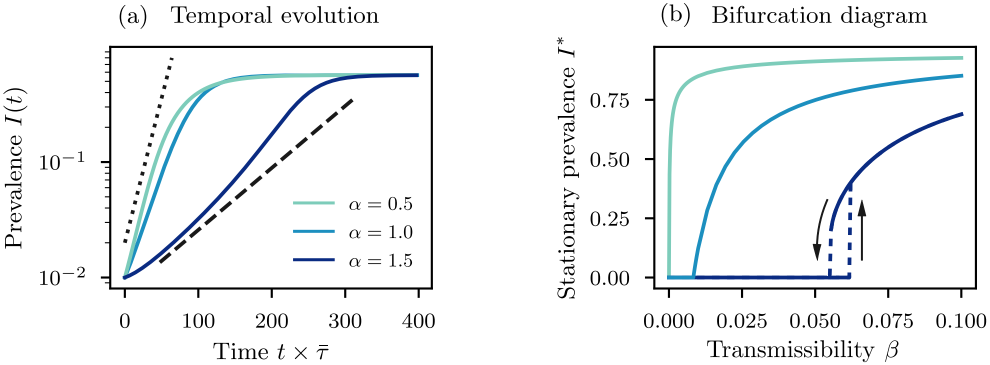
  LINEAR INFECTION KERNELS ARE THE EXCEPTION RATHER THAN THE NORM

Consequences of nonlinear infection kernel



Superexponential spread and discontinuous phase transition



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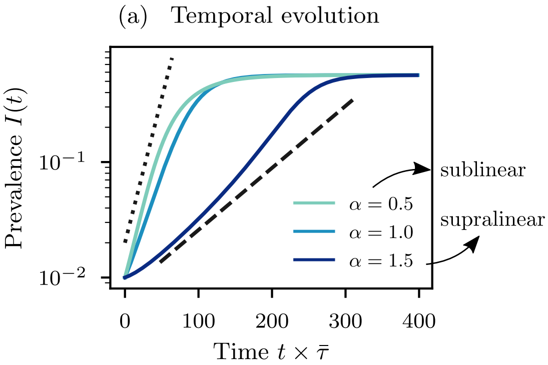
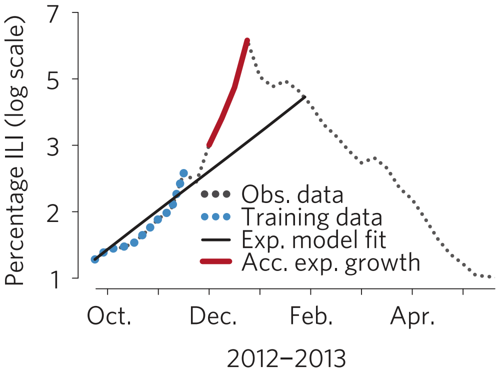


Superexponential spread of Influenza-Like-Illness [2]



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MAYBE WE SHOULDN'T, maybe we should adopt more general forms, e.g.,

$$\theta(\rho) \propto \rho^{\nu} \quad \text{with } \nu \in \mathbb{R}^{+}$$

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MAYBE WE SHOULDN'T, maybe we should adopt more general forms, e.g.,

$$\theta(\rho) \propto \rho^{\nu} \quad \text{with } \nu \in \mathbb{R}^{+}$$

For a standard SIR model, this could look like

$$\frac{\mathrm{d}S}{\mathrm{d}t} \approx -\beta S\, I^{\nu} \ .$$

Aknowledgments

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Sentinel North

Appendix

Mathematical description for $N \to \infty$

We track $\rho_k(t)$ the fraction of infected nodes of membership k using

$$\rho_k(t+1) = (1-\mu)\rho_k(t) + (1-\rho_k(t))\Theta_k \ ,$$

where

$$\Theta_k(\bar{\rho}) = 1 - [1-\bar{\theta}(\bar{\rho})]^k \ , \quad \bar{\rho}(t) = \sum_k \rho_k(t)\frac{k\tilde{P}(k)}{\langle k \rangle} \ , \quad \bar{\theta}(\bar{\rho}) = \sum_m \bar{\theta}_m(\bar{\rho})\frac{m\hat{P}(m)}{\langle m \rangle} \ ,$$

and

$$\bar{\theta}_m(\bar{\rho}) = \sum_{i=0}^{m-1} \binom{m-1}{i} \bar{\rho}^i (1-\bar{\rho})^{m-1-i} \, \theta_m\left(\frac{i}{m-1}\right) \ .$$