NETWORKS 2021

Universal nonlinear infection kernel from heterogeneous exposure on higher-order networks

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Standard epidemiological models predict exponential growth

For a whole population, with I the fraction of infectious,

$$\frac{\mathrm{d}I}{\mathrm{d}t} \approx \lambda\, I \qquad\qquad (I \ll 1)$$

$$\implies I \propto e^{\lambda t}$$

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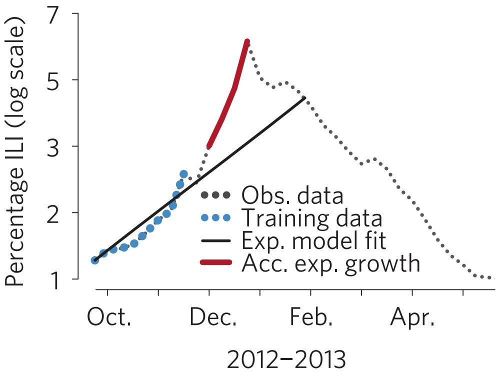
But this is because we assume that the risk of infection is *linear*

$$\theta(I) \propto I$$

Superexponential spreading of Influenza-Like-Illness



Scarpino, S. V., Allard, A., & Hébert-Dufresne, L. (2016). The effect of a prudent adaptive behaviour on disease transmission. Nature Physics, 12(11), 1042-1046.

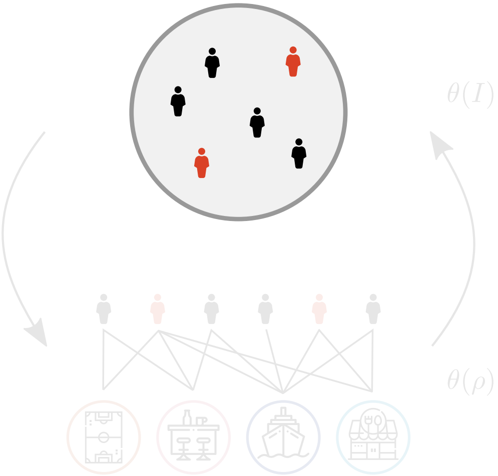
$$\theta(I) \propto I$$

(i) Why assume linearity?

(ii) When is linearity valid?

(iii) What other forms could it take?

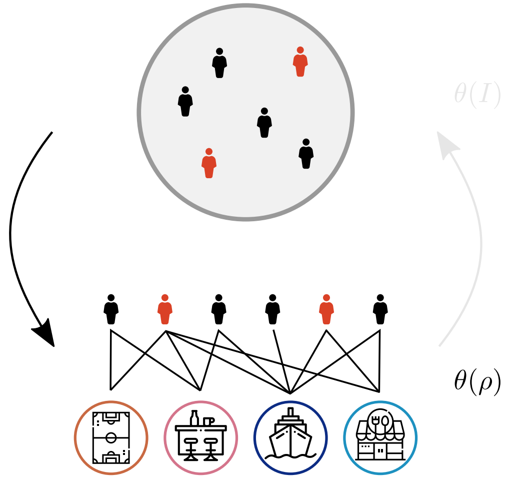
Population level



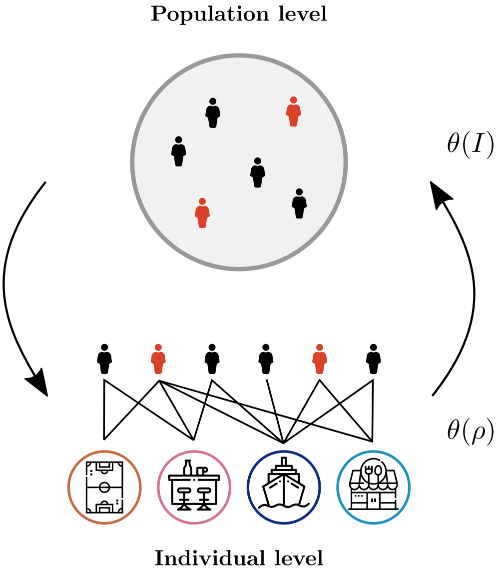
Individual level

Icons made by Freepik from www.flaticon.com

Population level



Individual level



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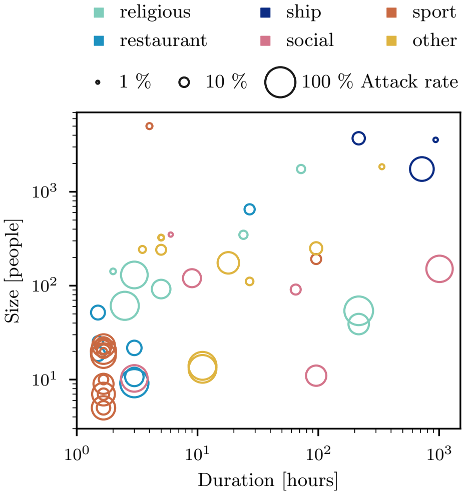
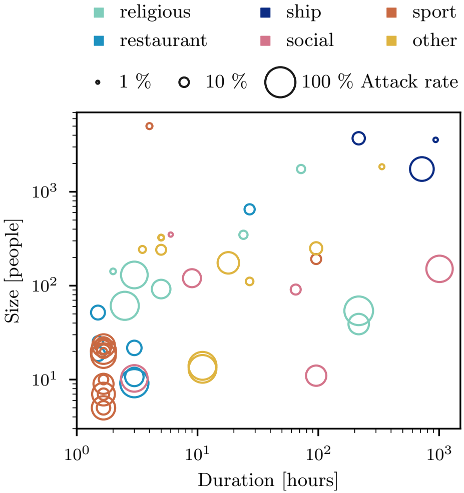


Model properties

1. Explicit group interactions in *environments*
2. Heterogeneous temporal patterns

St-Onge, G., Sun, H., Allard, A., Hébert-Dufresne, L., & Bianconi, G. (2021). Universal nonlinear infection kernel from heterogeneous exposure on higher-order networks. arXiv :2101.07229.

Motivation for the framework : superspreading events



Model properties

1. Explicit group interactions in *environments*
2. Heterogeneous temporal patterns
3. *Minimal infective dose* (threshold model)

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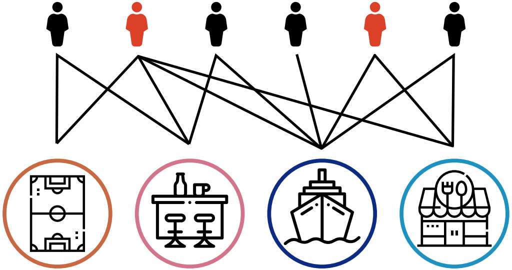
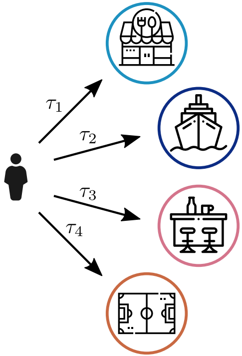
Property # 1 : Higher-order interactions – bipartite structure



Property # 2 : heterogeneous temporal patterns



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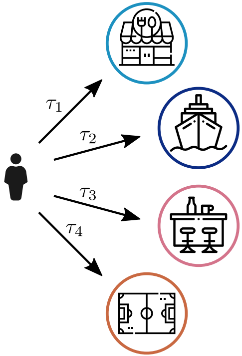

What should be $p(\tau)$?

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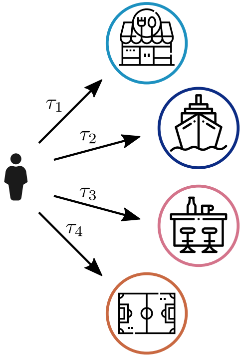
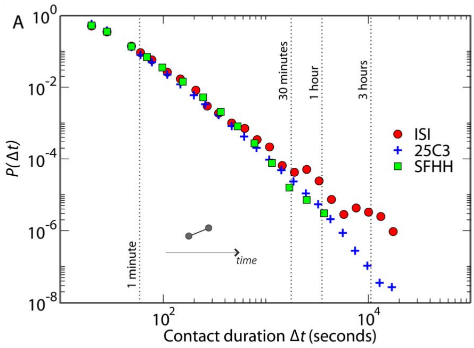


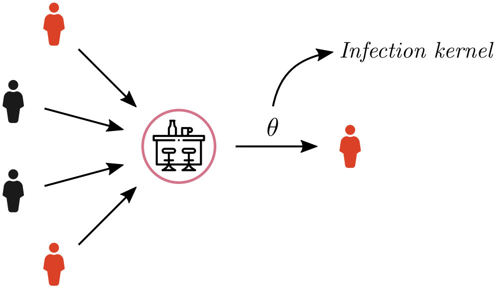


What should be $p(\tau)$?

$$p(\tau) \propto \tau^{-\alpha}$$

Cattuto, C., Van den Broeck, W., Barrat, A., Colizza, V., Pinton, J. F., & Vespignani, A. (2010). Dynamics of person-to-person interactions from distributed RFID sensor networks. PLOS ONE, 5(7), e11596.

Risk of infection in an environment

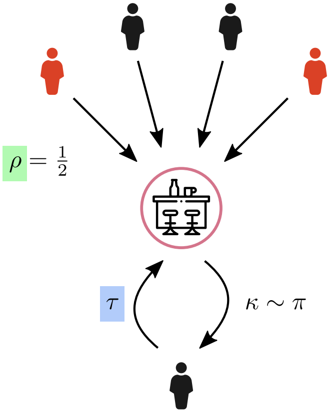


θ : probability of infection (per environment) during one time step

Property #3 : threshold model

- Individual receives a dose $\kappa \sim \pi(\kappa; \rho, \tau)$
- The fraction of infectious participants is ρ
- The mean dose received is

$$\langle \kappa \rangle \propto \rho\tau$$



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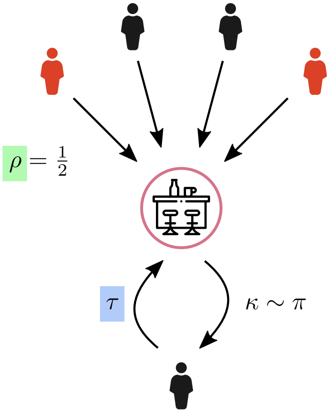
- Our immune system is able to fight mild challenges
- A minimal infective dose K is required for infection



Universal nonlinear infection kernel

The infection kernel is

$$\theta(\rho) = P(\kappa \geq K) = \int_1^T \int_0^K p(\tau)\pi(\kappa; \rho, \tau)\, \mathrm{d}\kappa\, \mathrm{d}\tau$$

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Assuming :

1. $p(\tau) \propto \tau^{-\alpha-1}$;
2. *Some technical conditions for the asymptotic analysis* ;

for a large class of dose distribution π, we recover the *universal* infection kernel

$$\boxed{\theta(\rho) \propto \rho^{\alpha}}$$

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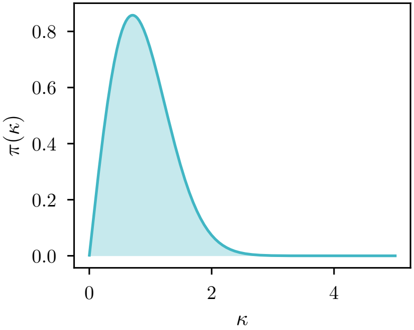
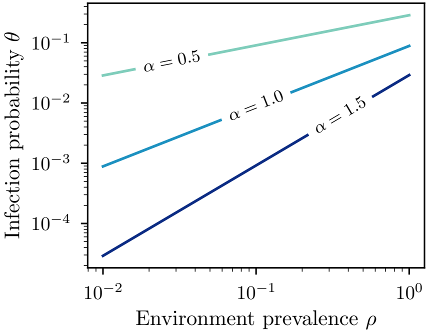
Weibull dose distribution

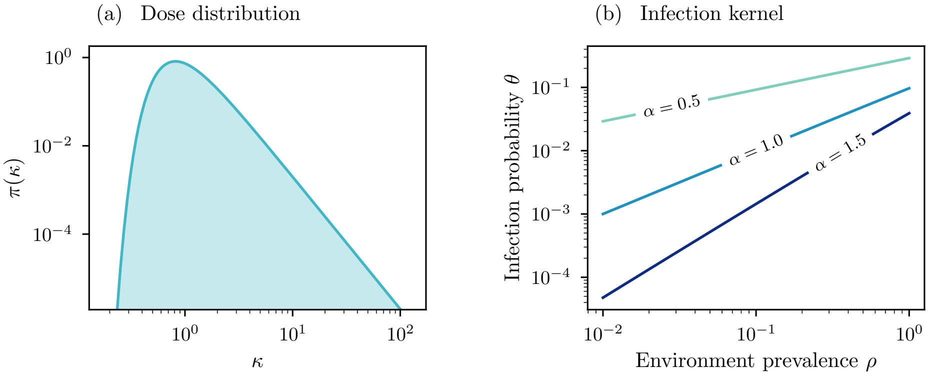
(a) Dose distribution



(b) Infection kernel



Fréchet dose distribution



When is linearity valid at the *individual level*?

- $\alpha = 1 \quad [P(\tau) \propto \tau^{-\alpha-1}]$
- π is a Poisson distribution and $K = 1$
- Some other limit cases

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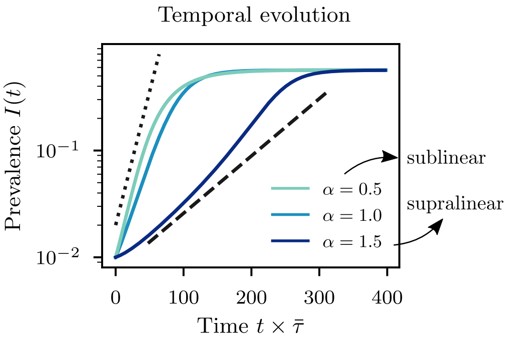
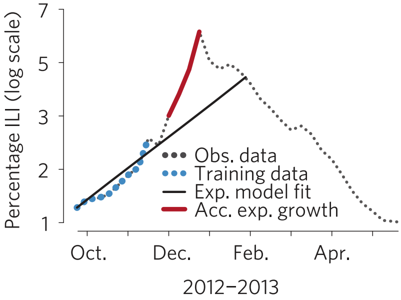
  LINEAR INFECTION KERNELS ARE THE <u>EXCEPTION</u> RATHER THAN THE NORM

Supralinear infection kernel lead to superexponential spreading





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Maybe we shouldn't, maybe we should adopt more general forms.

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At the *individual level*, we found

$$\theta(\rho) \propto \rho^{\alpha} \quad \text{with } \alpha \in \mathbb{R}^{+}$$

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At the *individual level*, we found

$$\theta(\rho) \propto \rho^\alpha \quad \text{with } \alpha \in \mathbb{R}^+$$

If we coarse grain at the *population level*,

$$\theta(I) \propto \begin{cases} I & \text{if } I \ll 1 \\ I^\alpha & \text{otherwise} \end{cases}$$

For a standard SIR model, this could look like

$$\frac{\mathrm{d}I}{\mathrm{d}t} \approx \lambda S\, \theta(I) - \mu I \ ,$$

Acknowledgments

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Hanlin Sun, Antoine Allard,
Laurent Hébert-Dufresne,
Ginestra Bianconi

Preprint available!

arXiv :2101.07229

SCAN ME

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Fonds de recherche Nature et technologies Québec

Calcul Québec

CANADA FIRST RESEARCH EXCELLENCE FUND | APOGÉE CANADA FONDS D'EXCELLENCE EN RECHERCHE

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