Geometric Evolution of Complex Networks

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In growing networks, preferential attachment (PA) is a probabilistic mechanism often argued to explain the scaling behaviour of the degree distribution of many real complex networks [1]. However, PA-based models lack the assortativity (or disassortativity) present in most complex networks [2], forcing us to consider other types of effective growth mechanisms. We present a novel type of growing model where new nodes arrive at time t and where the nodes with which they form new links are chosen *homogeneously* among all existing nodes, in sharp contrast with the PA prescription. We find that this homogeneous distribution of the links has an interesting geometric interpretation: it gives rise to growing geometric graphs with time-dependent Fermi-Dirac connection probability

$$Pr[i \leftrightarrow j,t] = \frac{1}{\exp\left[\frac{\beta}{2}\left(d_{ij} - \mu(t)\right)\right] + 1}$$

where $\mu(t)$ is a general time-dependent chemical potential function, β is an inverse temperature controlling the clustering coefficient and d_{ij} is the distance between nodes i and j embedded in an isotropic and homogeneous geometric space. Interestingly, this model is considerably more tractable than PA-like models in terms of calculating the structural properties of the generated networks. Moreover, we find that $\mu(t)$ can be chosen to fit a given degree distribution and that the order of appearance of the nodes in the network's evolution, its *history*, affects the structure of these networks at the level of the connection correlations. An effective history can then be inferred with a maximum likelihood estimation method to better describe a real network structure, allowing us to capture their assortative (disassortative) behaviour.

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[2] M. E. J. Newman, Phys. Rev. Lett. **89**, 208701 (2002).

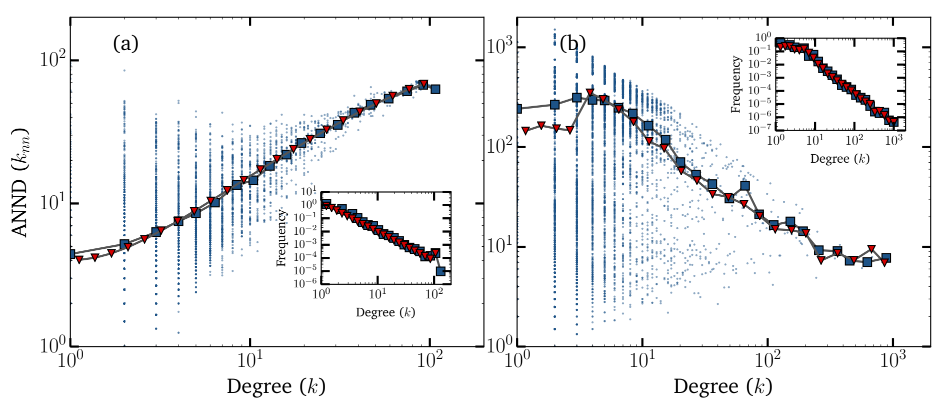

Figure 1: Average degree of nearest neighbors (ANND) k_{nn} as a function of the degree k of nodes for synthetic growing geometric graphs with scale-free degree distribution $P(k) \sim k^{-\gamma}$ and for two different histories: (a) ordering of the nodes according to their degree (new/old nodes have low/high degree), (b) random ordering of the nodes (no correlation between their degree and their ordering). We measure for (a) a degree assortativity coefficient $r \simeq 0.7$ (assortative regime) and for (b) $r \simeq -0.12$ (disassortative regime due to structural constraints). For these synthetic networks, the selected space is a circle of radius $R = \frac{N}{2\pi}$, the number of nodes $N = 10^4$, the average degree $\langle k \rangle = 6$, $\gamma = 2.2$ and $\beta = 100$. The small blue dots correspond to data points, the blue squares to logarithmically binned data points and the red triangles to analytic solutions. Inset: degree distribution of each network.