# Complex networks are an emerging property of hierarchical preferential attachment[1]

Laurent Hébert-Dufresne, Edward Laurence, Antoine Allard, Jean-Gabriel Young & Louis J. Dubé

*Département de Physique, de Génie Physique, et d'Optique, Université Laval, Québec, QC, Canada*

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Real complex systems are not rigidly structured; no clear rules or blueprints exist for their construction. Yet, amidst their apparent randomness, complex structural properties appear to universally emerge. We propose a hierarchical preferential attachment process (HPA) which illustrates how an important class of complex systems can be modelled as a construction of many levels of organization following a single universal growth principle. We give examples of such hierarchy in real systems, for instance in the pyramid of production entities of the movie industry. More importantly, we show how real complex networks can be interpreted as a projection of our model, from which their scale independence, their clustering, their hierarchy, their fractality and their navigability naturally emerge. Our results suggest that complex networks, viewed as growing systems, can be quite simple, and that the apparent complexity of their structure is largely a reflection of the complex hierarchical nature of our world.

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[1] Hébert-Dufresne *et al.*, arXiv:1312.0171

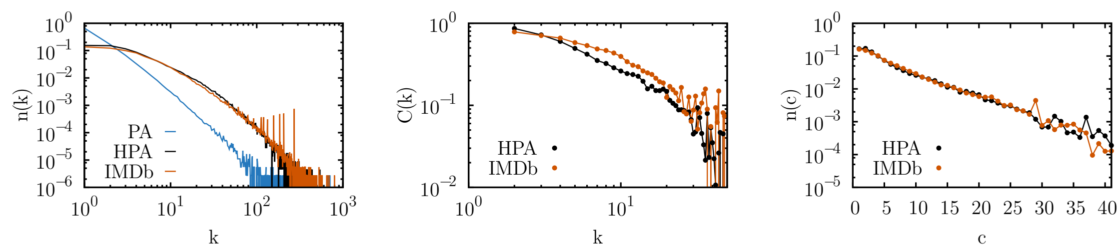


*Figure 1:* **Scale-independence and clustering of a projected hierarchical systems.** We reproduce the hierarchical production structure of the Internet Movie Database, by preferentially distributing producers in films, embedded in companies embedded in countries. The co-production network is obtained by projecting the full hierarchical system. (left) Degree distribution, a network obtained through the classic preferential attachment model (PA) is shown for comparison. (middle) Average clustering coefficient for nodes of degree $k$. The PA would give a vanishing clustering $C(k) = 0$ for all $k$. (right) Distribution of node centrality quantified by their coreness $c$ under $k$-core decomposition of the networks. The PA would lead to a unique shell of coreness $c = 1$ because of the network's tree like structure.

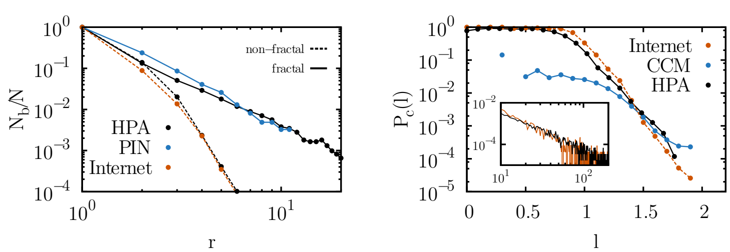


*Figure 2:* **Fractality and naviguability of projected hierarchical systems.** (left) Box counting results on a well-known fractal network (protein interaction network of Homo Sapiens) and a non-fractal network (the Internet at the level of autonomous systems). HPA can reproduce both behaviour by controlling the mixing between different hierarchical levels. (right) Probability of connection $P_c(l)$ between nodes a distance $l$ apart after an inferred projection of the networks unto an hyperbolic space (inset: distribution $n(k)$ versus degree $k$). The CCM corresponds to a rewired network preserving degree distribution and degree-degree correlations, but obviously lacking the more complex structural correlations.