Phase space engineering in optical microcavities

II. Controlling the far field

Julien Poirier, Guillaume Painchaud-April, Denis Gagnon, Louis J. Dubé

Département de Physique, de Génie Physique, et d'Optique, Université Laval, Québec, Canada

Context

Optical microcavities of regular shape (disk, toroid, sphere) are known to give rise to high quality resonances, the so-called Whispering Gallery Modes (WGMs). However, these modes display a uniform intensity distribution both in the near-field (NF) and the far-field (FF). Geometric perturbation alone of these cavities (e.g. circular to quadrupolar) can lead to directional FF emission, but this is generally associated with an important loss in Quality factor. Many applications, such as microlasers, require both a directional FF emission and a high Q-factor.

*We present a method to achieve this apparently conflicting goal on an annular cavity (**Fig. 1**). Emphasis is given on the control and the prediction of the FF profile.*

Annular Cavity

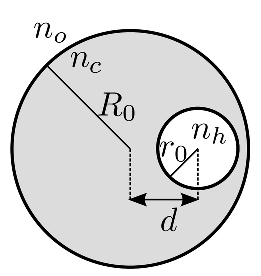
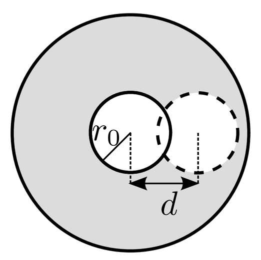


Fig. 1: The annular cavity.

Our system's configuration is a dielectric annular cavity: a circular cavity of radius R_0 and refractive index n_c, surrounded by a medium of index n_o, with a circular inclusion (hole) of radius r_0 and index n_h displaced a distance d from the cavity center. For the sake of the presentation, some numerical parameters are fixed at nominal values: $R_0 = 1$, $n_o = n_h = 1$ and $n_c = 3.2$. The two remaining variables $(d; r_0)$ will serve as control parameters.

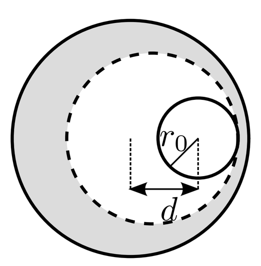
Scenario 1: Inducing directional FF emission



It has been shown elsewhere [1] that an eccentric inclusion ($d \neq 0$) and an appropriate choice of d and r_0 can induce a directional FF emission while preserving the NF character (and therefore a high Q-factor) of WGMs.

For more details, see the companion presentation to this poster:
• Session MPM III, Tu.C4.6

Scenario 2: Controlling the FF emission

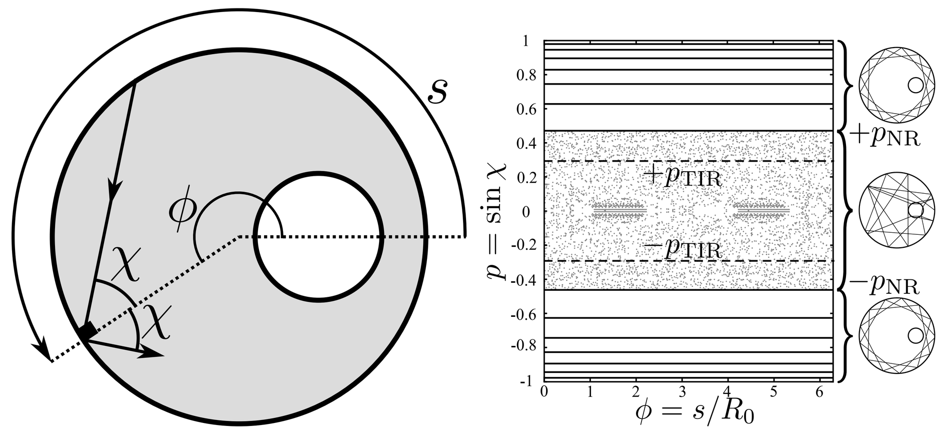


Having chosen a directional mode with Scenario 1, the FF directions may be controlled by keeping the group $d + r_0$ constant and changing r_0. The phase-space engineering associated with this approach (Scenario 2) and its effects on the FF is the subject of this presentation.

The Classical Phase Space

The classical dynamics on the annular cavity possess interesting and almost unique characteristics

- ***Poincaré map*** *on the cavity boundary* $\mathcal{P} : (s_i, p_i = \sin\chi_i) \mapsto (s_{i+1}, p_{i+1})$
- ***Well separated mixed phase space:***
  - *Non-Regular region for* $|p| < p_{NR} = (d + r_0)/R_0$
  - *Regular region for* $|p| \geq p_{NR}$
- ***Emission region*** *bounded by Total Internal Reflection (TIR)* $E = \{(s, p) \ : 0 \leq s \leq 2\pi R_0, |p| \leq p_{\text{TIR}} = n_o/n_c\}$

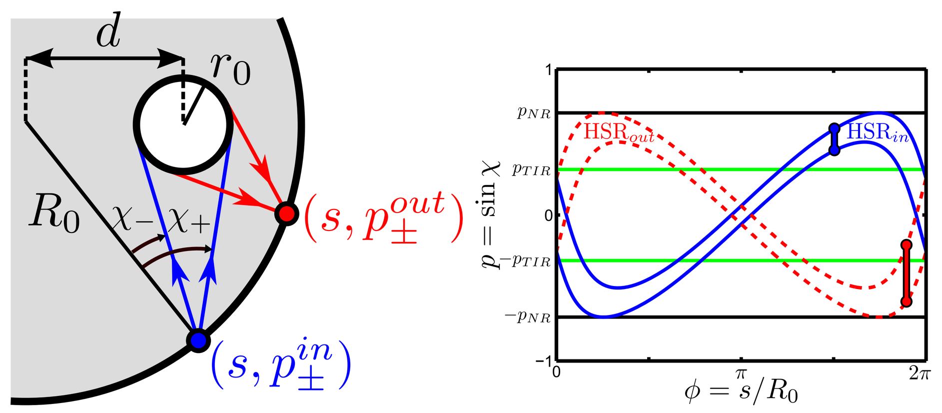


Hole Scattering Region (HSR)

- *Together, these regions define the mixing properties*
- *HSR$_{out}$ gives rise to an effective emission region originating from HSR$_{in}$*

$$W = \mathcal{P}(\mathcal{P}^{-1}(E) \cap \bar{E}), \tag{1}$$

with $\bar{E}$ being the complement of E.



Ray escape

1. *Initials conditions: $\{s^i, p^i, I^i = 1\}$ are given by the Husimi distribution [2] of the unperturbed modes inside HSR$_{in}$.*
2. *Ray-splitting dynamics: Rays are allowed to split as they interact with the inclusion. Intensities (reflected and transmitted) are calculated according to the Fresnel coefficients.*
3. *FF escape: For each interaction with the cavity boundary, the Fresnel coefficients generalized for curved interfaces [3] determine the loss in intensity. This gives rise to a set of escaping rays $\{s^i_j, p^i_j, I^i_j, \theta^i_j\}$.*
4. *Classical emission distribution equivalent to the Husimi distribution:*

$$H^{class}(s, p) \propto \sum_{i,j} I^i_j G(s; s^i_j) G(p; p^i_j), \tag{2}$$

where $G(a; b)$ is a Gaussian function centered at a and evaluated at b with a dispersion equal to the one of the Husimi distribution.

The Wave Equation

The physical problem of interest reduces to solving the 2D Helmholtz equation

$$\left[\nabla^2 + n^2(\mathbf{r})k^2\right] \psi(\mathbf{r}) = 0. \tag{3}$$

Outside of the cavity ($r \geq R_0$), the solutions are expanded in an angular basis

$$\psi(\mathbf{r}) = \sum_{m=-\infty}^{\infty} \left[A_m H^{(2)}_m(n_o kr) + B_m H^{(1)}_m(n_o kr)\right] e^{im\phi}, \tag{4}$$

with $H^{(1,2)}_m(\cdot)$ being the Hankel functions and A_m (B_m) the incoming (outgoing) wave coefficients. For an unperturbed mode (no inclusion), only one component, say m_0, is present.

Contrast measure

In order to characterize the emission profile, a contrast measure C_{m_0} is defined as

$$C_{m_0}(r) = \frac{\sum_{|m| \neq m_0} \left|B_m H^{(1)}_m(n_o kr)\right|^2}{\sum_m \left|B_m H^{(1)}_m(n_o kr)\right|^2}. \tag{5}$$

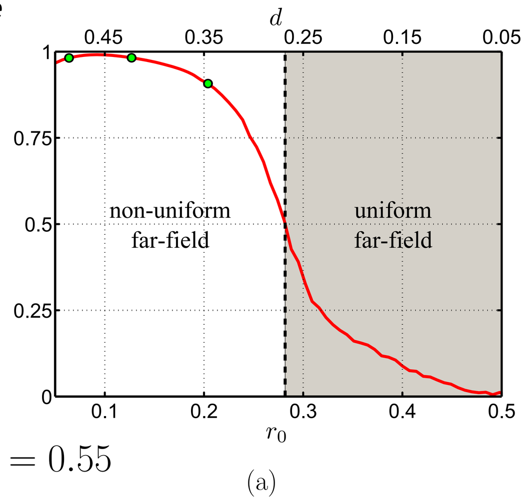
Two interesting limits

- *Near field : $C_{m_0}(r = R_0) = 0 \Rightarrow$ high Q-factor.*
- *Far field : $C_{m_0}(r \to \infty) = 1 \Rightarrow$ directional emission profile.*

Classical/Wave Results

For demonstration, one mode has been selected to present evidence of FF modulations

- *TM$_{(m=11;n=1)}$, m and n azimuthal and radial mode number respectively*
- $kR_0 \sim 4.5$
- Even and odd symmetries
- $r_0 \in [0.05,\ 0.5]$, with $d + r_0 = 0.55$



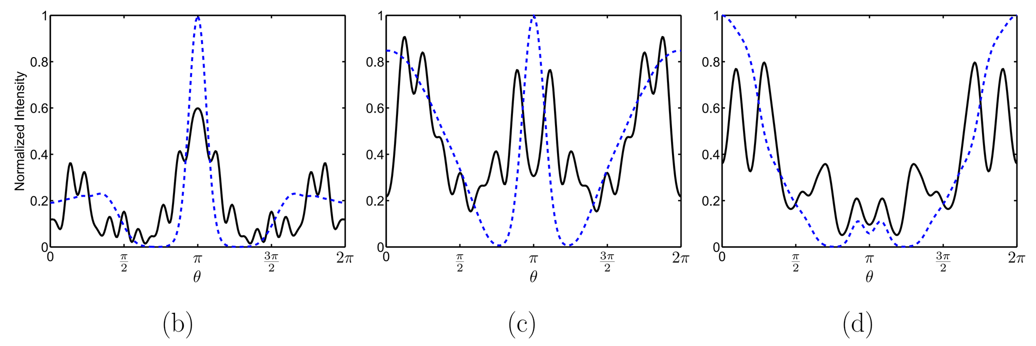


Fig. 2: ***(a) FF contrast measure**. A large range of parameters leads to a non-uniform FF, arbitrarily defined as $C_{11} > 0.5$. **(b)-(d) FF profiles for 3 parameter values**: $r_0 = 0.064R_0$, $0.127R_0$ and $0.211R_0$. The full curve represents the combined envelope of the two symmetries $|\psi^e_{(11,1)}|^2 + |\psi^o_{(11,1)}|^2$, while the dashed line displays the classical FF profile obtained by ray escape using initials conditions near p_{NR}.*

Correspondence

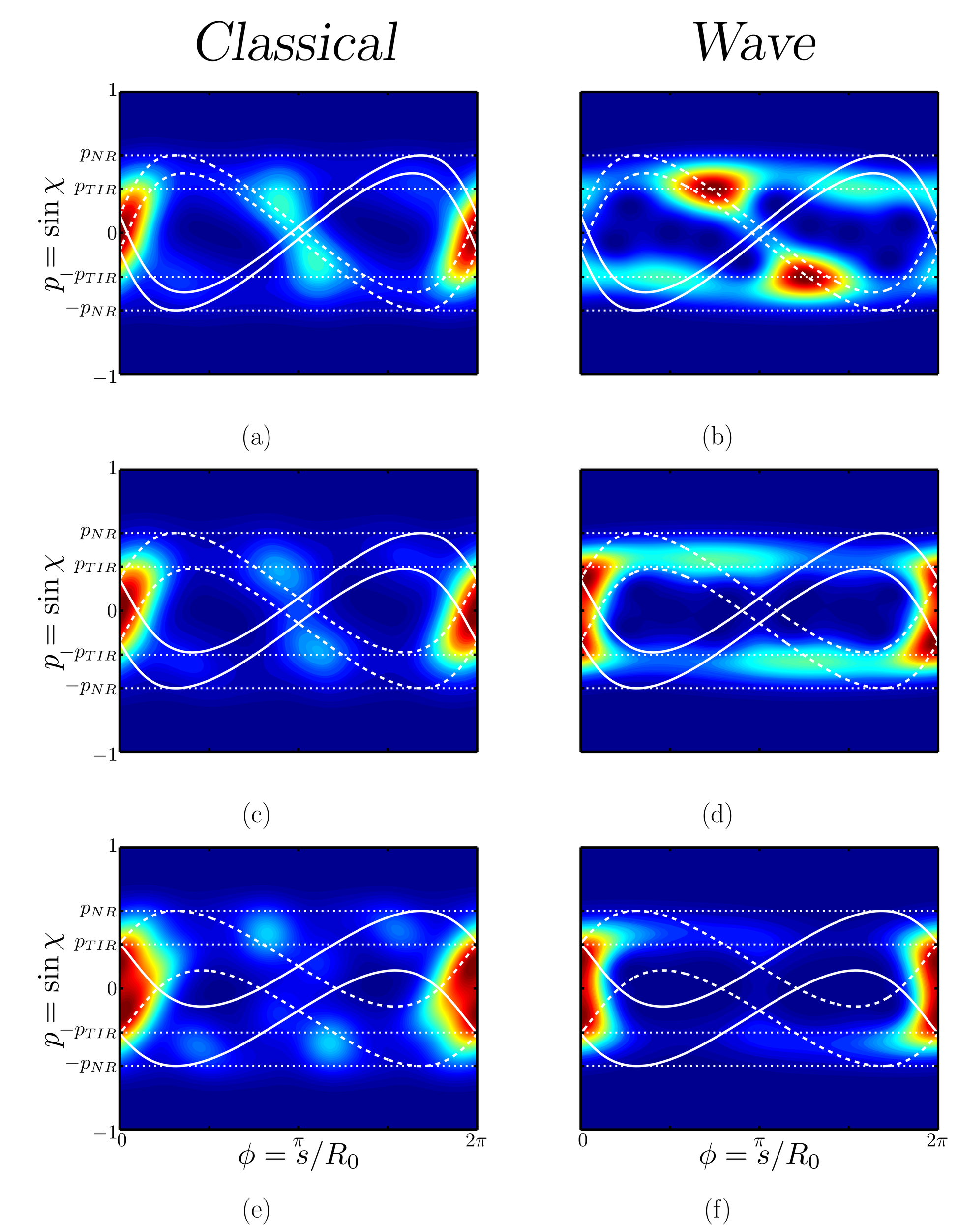


Fig. 3: ***Left column: classical emission distributions** obtained using 250,000 starting points. **Right column: Husimi distributions** of the Non-Regular outgoing part of the even mode ψ^e_{NR}. Each row is for different sets of parameters: from top to bottom $r_0 = 0.064R_0$, $0.127R_0$ and $0.211R_0$ with $d + r_0$ fixed at 0.55.*

- *The NR outgoing part of the even mode* $\psi^e_{NR} = \sum_{|m| < n_c kR_0 p_{NR}} B_m H^{(1)}_m(n_o kr)e^{im\phi}$.
- *Maximum intensity is always located inside HSR$_{out}$.*
- *Good agreement between classical and wave calculations, except for first row where the diffractive limit: $2r_0 \ll \lambda/n_c$ is not respected.*

Conclusion

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- *Control of the directionality of the FF emission in an annular dielectric cavity is feasible*
- *FF profiles of both full-wave and classical simulations show similar structures*
- *This work opens the way to the phase space design scenarios for high-Q directional emission*
- *This method can be generalized to any inclusion shapes, thereby enabling FF customization.*

[1] G. Painchaud-April *et al.*, submitted to *Phys. Rev. E.* (2010).
[2] Lee, H.-W, *Phys. Rep.* 259, 147-211 (1995).
[3] Hentschel, M. and Schomerus, *Phys. Rev. E* 65, 045603 (2002).