Functional resilience in dynamical complex networks with adaptive connectivity

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Summary

The brain is a notoriously resilient system. Although dynamical effects on the brain activity resulting from failures of its network have been found [1], most studies about resilient complex systems have so far focused on purely topological properties.

- We present a model of dynamics on network with connectivity adaptation to study resilience of neural networks.
- An effective formalism accurately describes the functional and structural states of neural networks.
- New resilience patterns emerge from the recovery of the system.

Model

Consider a graph composed of n nodes (neurons) and m directed edges (synapses). At time t, node i has activity $y_i(t)$ while the weight of the edge from j to i is $w_{ij}(t)$.

Firing-rate model

The activity of each node evolves according to the firing-rate model:

$$\dot{y}_i = \tau_N^{-1}\left(-y_i + \alpha\sigma\left[\lambda\left(\textstyle\sum_j w_{ij}y_j - \mu\right)\right]\right) \quad ; \quad \sigma(y) = \frac{1}{1+\mathrm{e}^{-y}}$$

Adaptive connectivity dynamics

Each excitatory edge can adapt according to **Hebb's rule with saturation**:

$$\dot{w}_{ij} = \tau_S^{-1}\left(\sigma_i\sigma_j - \gamma w_{ij}\sigma_j^2\right) \quad ; \quad \sigma_i = \sigma\left[\lambda\left(\textstyle\sum_j w_{ij}y_j - \mu\right)\right]$$

- Weights are bounded
- Weights deteriorate if inactive
- The ratio τ_N/τ_S will prove to be an important parameter.

Effective formalism

In 2016, Ref. [2] proposed an effective formalism to describe the dynamics of a network under perturbations.

- Unidimensional description of N-dimensional systems.
- Single focal node description and single effective structural parameter.

We define the input activity $x_i(t)$ of node i as $x_i(t) = \sum_j w_{ij}(t)y_j(t)$

We obtain an effective description of neural dynamics.

$$\dot{x}_{\mathrm{eff}} = \tau_N^{-1}\left(-x_{\mathrm{eff}} + \alpha\beta_{\mathrm{eff}}\sigma\left[\lambda(x_{\mathrm{eff}} - \mu)\right]\right)$$

$$x_{\mathrm{eff}} = \frac{\sum_{ij} w_{ij}x_j}{\sum_{ij} w_{ij}}$$

$$\beta_{\mathrm{eff}} = \frac{\sum_{ijk} w_{ij}w_{jk}}{\sum_{ij} w_{ij}}$$

Validity of the approximation:

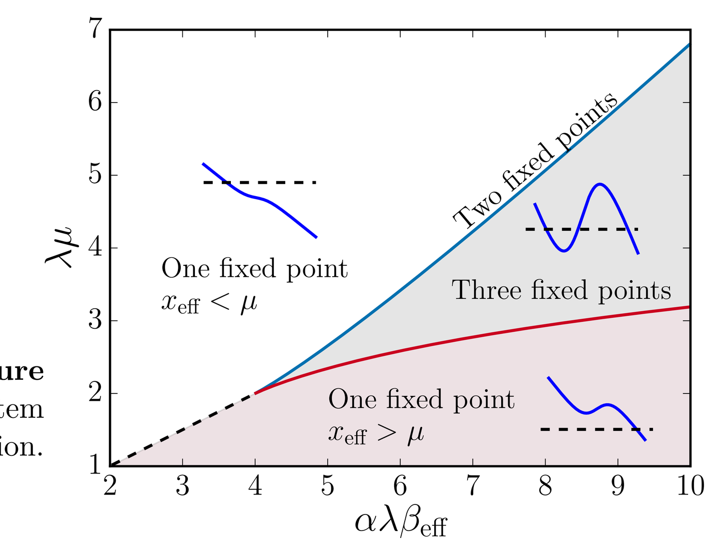
- Excitatory dynamics
- Homogeneous networks
- Directed and weighted networks

Resilience of networks

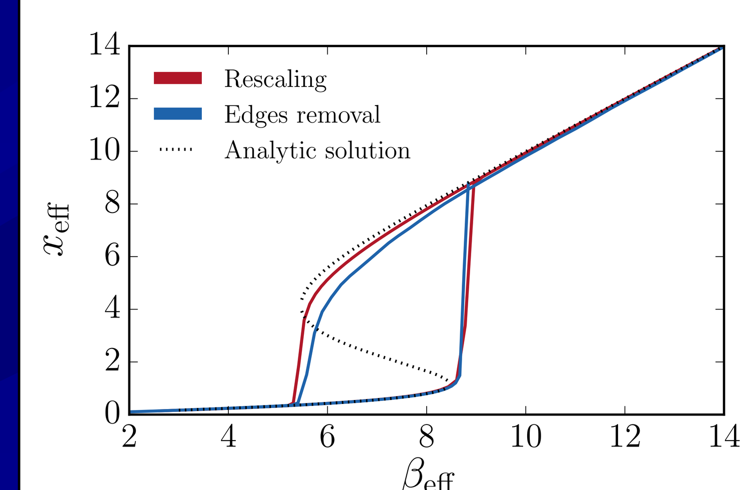
Stability analysis

We study the stability of the effective system.

- There is at least one fixed point and at most three fixed points.
- Three fixed points emerge at $\alpha\lambda\beta_{\mathrm{eff}} > 4$ and $\lambda\mu > 2$.



Right figure
Bifurcation diagram for the dynamics system without adaptation.



Top figure
Comparison of the analytical solution and numerical simulations of the dynamics on random graphs of size $n = 100$ and density $p = 0.2$. During a rescaling attack, each edge's weight is rescaled. On edge removal, a fraction of edges are removed from the graph.

Hysteresis

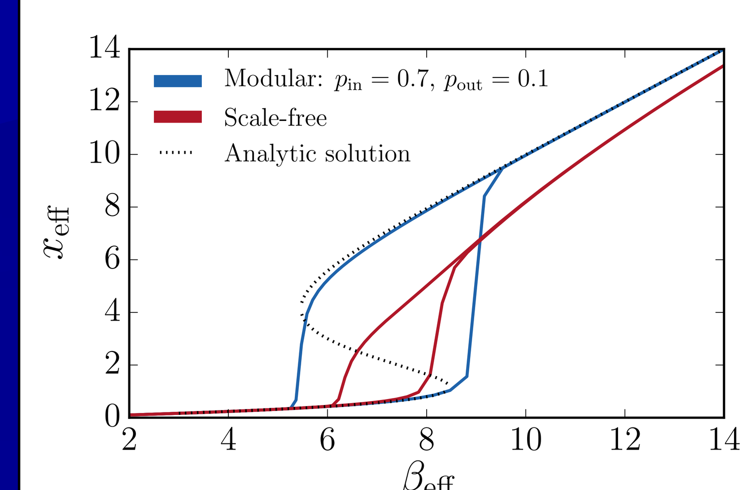
For a given set of parameters, the system shows a hysteresis region with three fixed points, two of which are stable.

- Analytical description of the hysteresis region.
- Effective description independent of the attack strategy.

Validity of the approximation

The effective formalism approximates the network to an effective node. We have tested this approximation for different structures.

- Excellent fit for homogeneous connectivity.
- Poor approximation for heteregenous graphs.



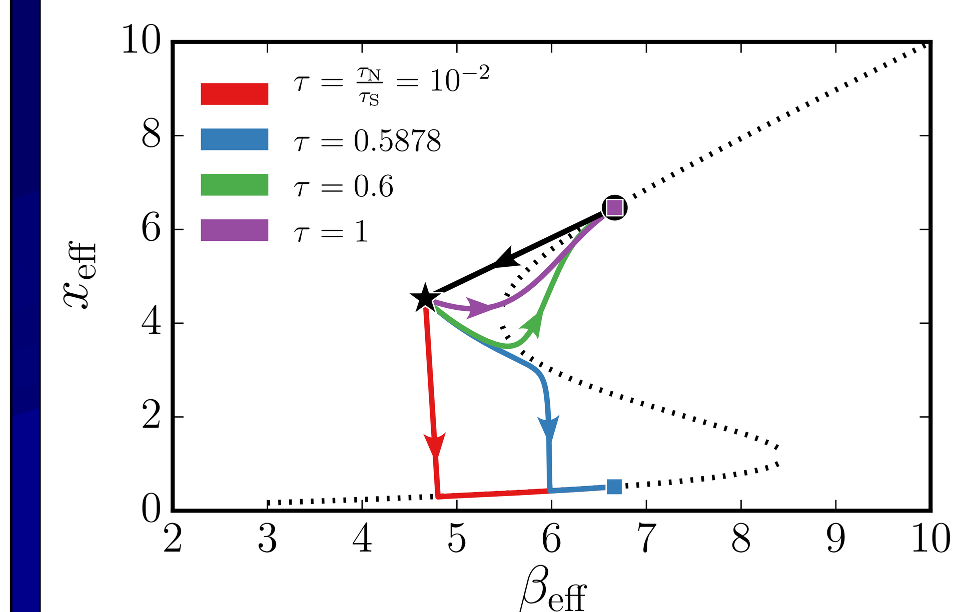
Left figure
Comparison of the effective approximation for networks of size $n = 100$ with modular structure and scale-free degree distribution.

Adaptive connectivity

Critical perturbation

After an attack, we let the system recover using its adaptive connectivity rule.

- Attacks result in a change of β_{eff}.
- β_{eff} is driven by the adaptive connectivity until a steady-state is reached.
- Loss of resilience happens if the system is unable to recover its initial activity x_{eff} and structure β_{eff}.



- **Ratio of characteristic times influences greatly the recovery results.**

Left figure
Influence of the ratio of characteristic times. An initial state (black circles) is attacked (black star). Then, the system recovers until steady-state is reached (squares).

Future works

- To quantifiy the resilience using our formalism. Promising candidates are:
  - Recovery time
  - Size of the hysteresis region
  - Attractiveness of the fixed points
- To obtain an effective and analytical description of the recovery process.
- To include inhibition in our model. To do so, we need to
  - Define an adaptive connectivity rule for inhibitory neurons
  - Extend the formalism to competitive dynamics

References

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