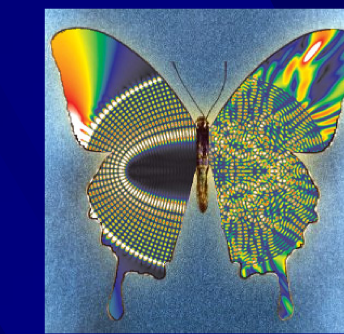
Wave Chaos in a New Class of Optical Microcavity

G. Painchaud-April [1], J. Poirier [1], P.-Y. St-Louis [1], J. Lépine [1], S. Saïdi [2], L.J. Dubé [1,2]

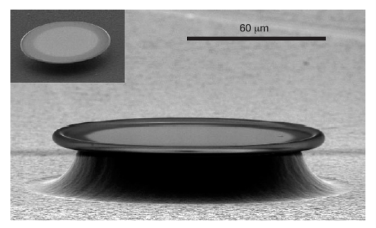
[1] Département de physique, de génie physique, et d'optique
Université Laval, Québec, Canada

[2] Laboratoire de chimie-physique – Matière et rayonnement
Université Pierre et Marie Curie, Paris, France



I. Introduction
Context

- modern electro-optics demands miniaturization of components
  - micro-lasers (and micro-resonators)
- problem:
  - Fabry-Pérot resonators hard to manufacture at µm scales
- possible solutions:
  - VErtical Cavity Surface Emitting Lasers (VECSELs)
  - dielectric micro-resonators
    - total internal reflection principle
    - classical counterpart (billiard) may be chaotic
    - interesting theoretical model for wave chaos studies

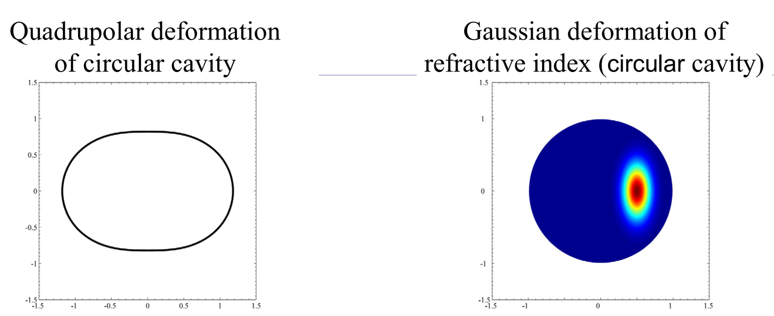


Toroid microcavity [1]

[1] D. K. Armani, T. J. Kippenberg, S. M. Spillane, and K. J. Vahala, Nature 421 (2003), 925-929.

I. Introduction
Cavity engineering

- disc cavity: isotropic emission
- loss of integrability in classical billiard systems
  - ⇒ emission directionality
- achieved through either/or
  - geometrical deformation of boundary
  - deformation of refractive index



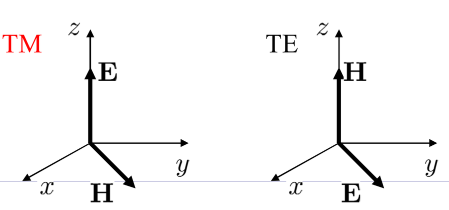
II. Numerical Method
Problem description

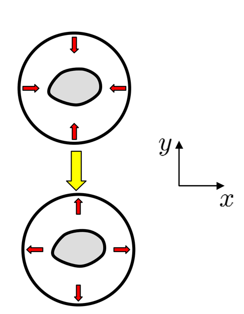
- optically thin resonator:
  - approximately 2D
- TM and TE polarization
  - Helmholtz homogeneous equation

$$\nabla^2\psi(\mathbf{r}) + n^2(\mathbf{r})k^2\psi(\mathbf{r}) = 0$$

$$\nabla n(\mathbf{r}) \neq 0$$

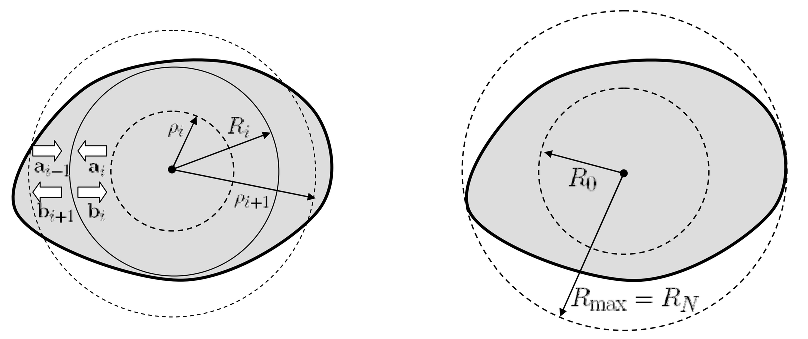
- time independent scattering approach
  - real wavelength
  - continuous incoming wave from infinity
  - detection at infinity
  - measurement of energy trapped in cavity (measurement of time delay)





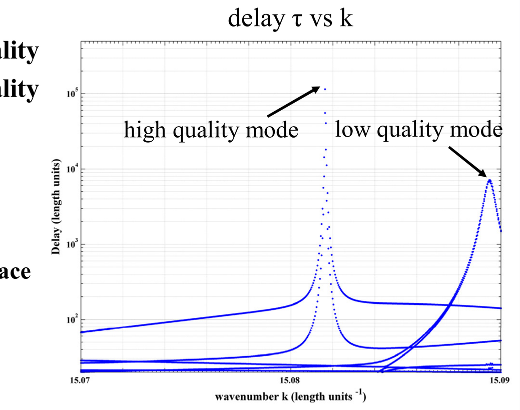
II. Numerical Method
Numerical calculation

- building the scattering matrix:
  - local development on thin rings
  - iterative construction of development coefficients
  - electromagnetic boundary conditions at each interface
  - connection to free space solution at R_N



II. Numerical Method
Results

- cavity response:
  - sharp resonances / high delay times τ / high quality modes Q = $\omega\,\tau$
  - directional emission at far-field
  - coupling between modes [2]:
    - high quality, low directionality
    - low quality, high directionality
- ray-wave correspondence:
  - Husimi distribution = projection of wave solution from coordinate space to corresponding classical phase space



[2] J. Wiersig and M. Hentschel, Phys. Rev. A, 73 (2006), p. 031802.

III. Geometrical Deformation
Standard approach

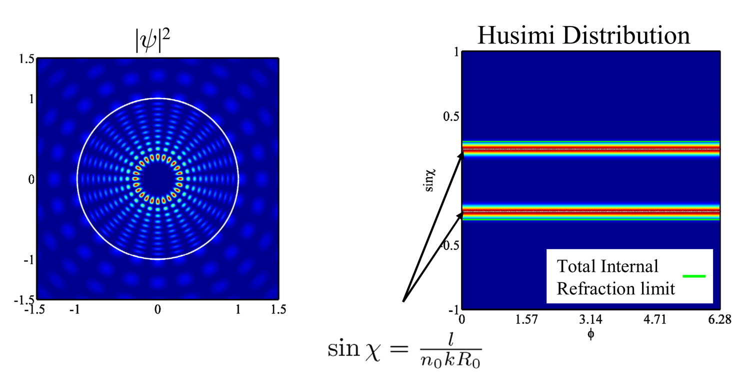
- deformation of the circle
  - circular to quadrupolar geometry

$$r(\phi) = R_0(1 + \epsilon\cos 2\phi)$$

  - control parameter: ϵ
    - ϵ = 0: classical integrable case
    - ϵ > 0: regular and chaotic dynamics (mixed phase space)
  - undeformed cavity radius: R_0 = 1
  - homogeneous medium
    - outside cavity: n_{out} = 1
    - inside cavity: n_0 = 3.3

III. Geometrical Deformation
A. Circular geometry (ϵ = 0)

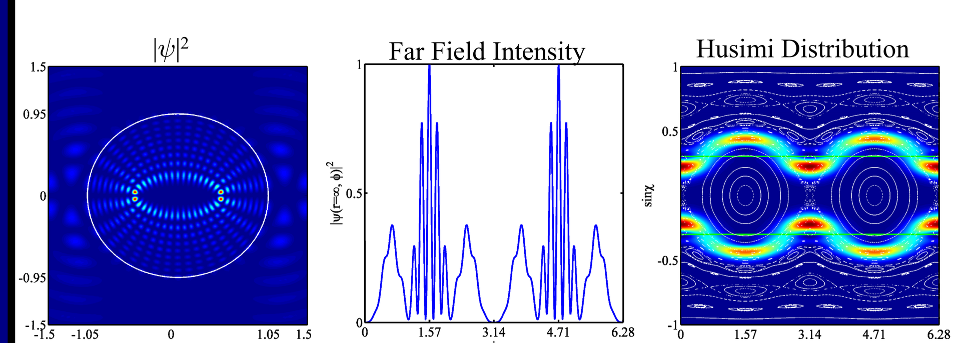
- sin χ: constant of motion
- regular phase space
- uniform far field intensity



$$\sin\chi = \frac{l}{n_0 k R_0}$$

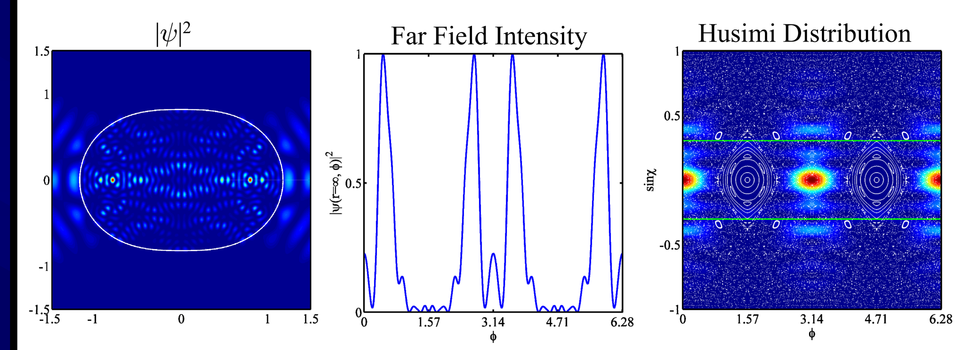
III. Geometrical Deformation
B. Quadrupolar geometry (ϵ = 0.05)

- low deformation



III. Geometrical Deformation
C. Quadrupolar geometry (ϵ = 0.18)

- high deformation



IV. Medium Deformation
New approach

- circular geometry with inhomogeneous medium
  - gaussian deformation

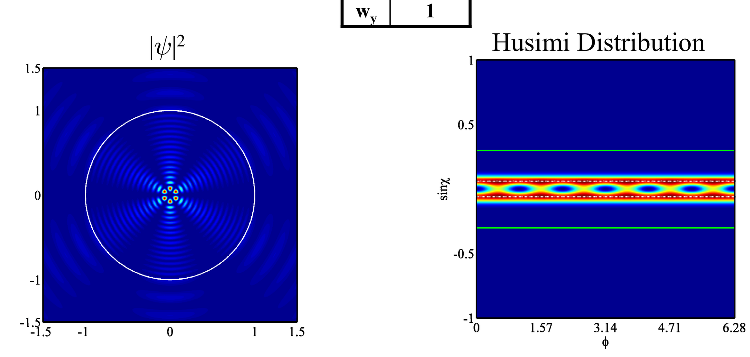
$$n(x,y) = n_0 + \delta n \exp\left[-2\frac{(x-x_0)^2}{w_x^2} - 2\frac{(y-y_0)^2}{w_y^2}\right]$$

  - integrable case, $x_0 = y_0 = 0$ and $w_x = w_y$, is analog to the circular homogeneous cavity
  - transition to chaos with loss of rotation symmetry using $w_x \neq w_y$, $x_0 \neq 0$ and/or $y_0 \neq 0$, with $\delta n \neq 0$

IV. Medium Deformation
A. Inhomogeneous integrable case

- sin χ: constant of motion
- regular phase space
- uniform far field intensity

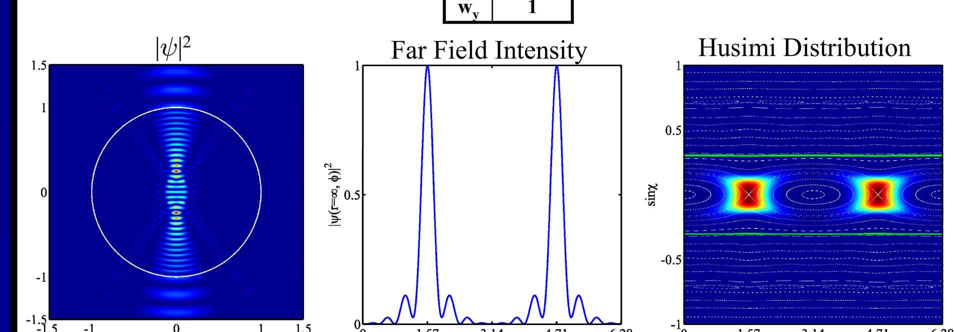
n_0	3.3
δn	0.33
x_0	0
y_0	0
w_x	1
w_y	1



IV. Medium Deformation
B. Inhomogeneous non-integrable case

- low deformation

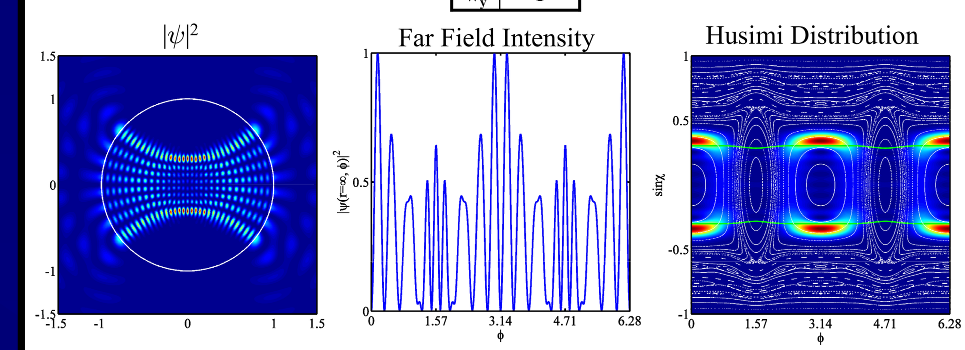
n_0	3.3
δn	0.165
x_0	0
y_0	0
w_x	0.5
w_y	1



IV. Medium Deformation
C. Inhomogeneous non-integrable case

- high deformation

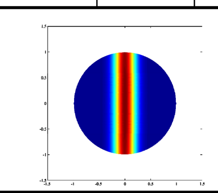
n_0	3.3
δn	1.65
x_0	0
y_0	0
w_x	0.5
w_y	1

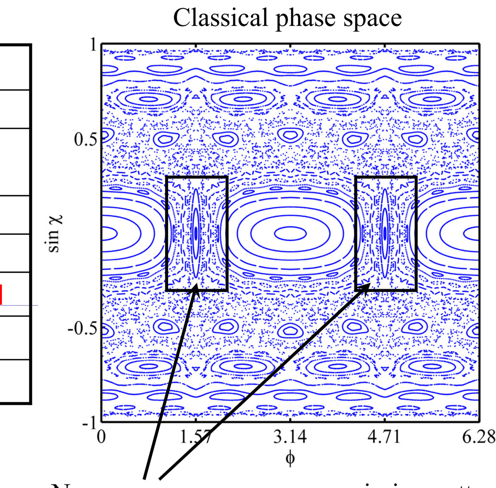


V. Phase Space Engineering
A plausible scenario

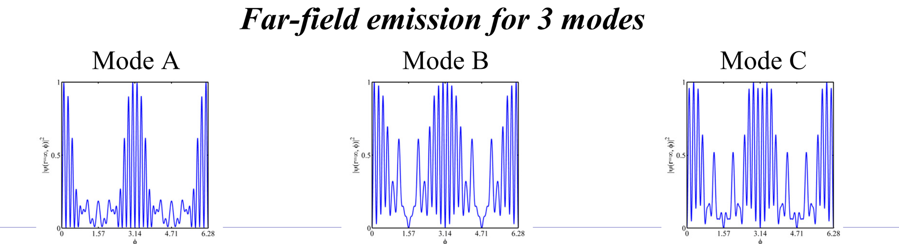
- parameters:

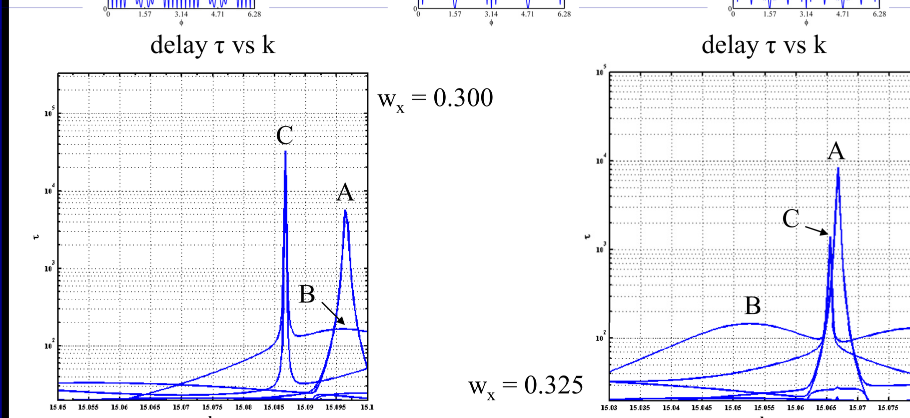
Exterior index	n_{out}	1
Interior index	n_0	3.3
Gaussian amplitude	δn	0.33
Cavity radius	R_0	1
Width, Y	w_y	8
Width, X	w_x	[0.3, 0.325]
Gaussian center Y	y_0	0
Gaussian center X	x_0	0



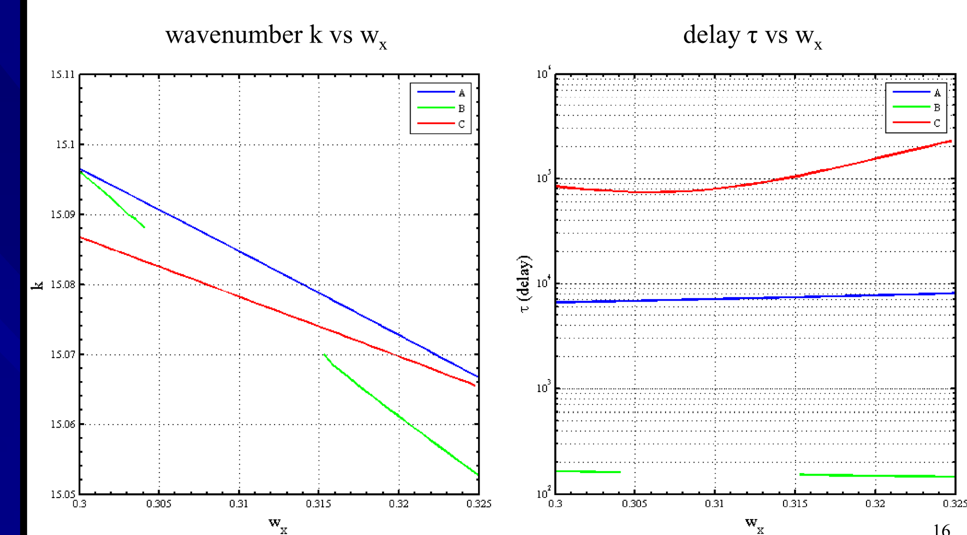


V. Phase Space Engineering
Far-field emission for 3 modes



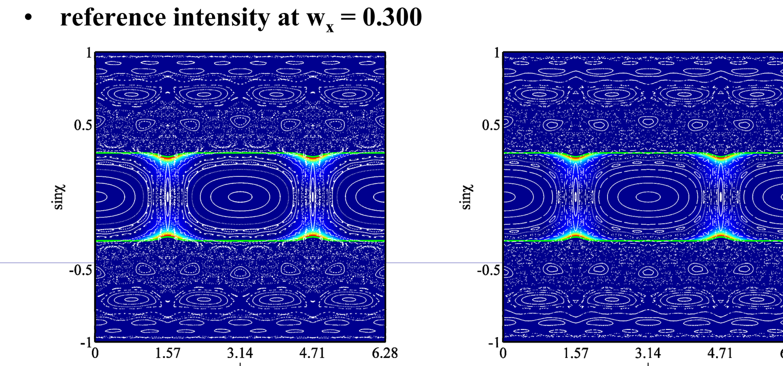


V. Phase Space Engineering
Mode interactions with parameter



V. Phase Space Engineering
Mode B Husimi distribution in escape region

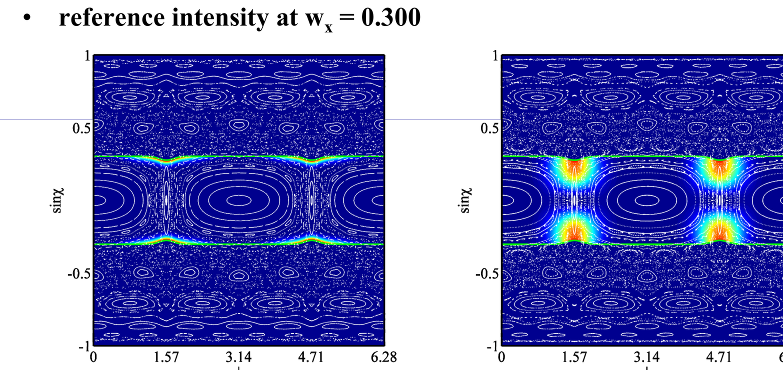
- reference intensity at w_x = 0.300



no obvious modifications of intensity in escape region

V. Phase Space Engineering
Mode C Husimi distribution in escape region

- reference intensity at w_x = 0.300



coupling starting near minimum of delay

⇒ modifications of intensity in escape region

VI. Summary

- introduction of a new method for symmetry breaking i.e. loss of integrability
- novel numerical method for (resonant) wave solutions
- comparison of geometrical and refractive index deformation effects on modes behaviour
- engineering phase space to obtain specific emission patterns
- coupling between modes can induce modifications of escape intensity
- future developments may include
  - non-linear studies (micro-lasers)
  - other index deformations
  - …