Classical Chaos in a Novel Inhomogeneous Photonic Billiard

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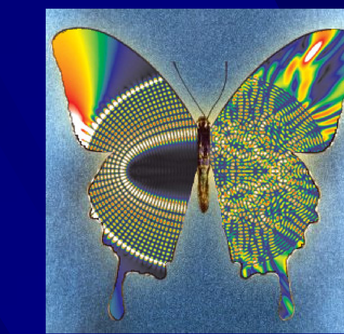
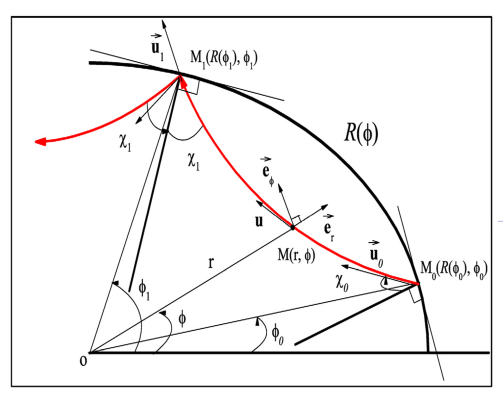


I. Introduction

- transition to chaos in an integrable billiard geometry
- chaos is induced NOT by a deformation of the geometry, but by the introduction of an inhomogeneous medium (e.g. a space-dependent refraction index)
- presentation of different scenarios for breaking the rotational symmetry of the cavity and inducing chaotic behaviour
- choice of a gaussian perturbation of the index of refraction
  - we study the consequences of this choice
  - we isolate the conditions of integrability
  - we describe the transition to chaos
  - we classify the effects of symmetry of the inhomogeneous medium on the trajectories

II. The Inhomogeneous Billiard Dynamics
The geometry

motion of a particle in a closed cavity $R(\phi)$
with specular reflexion at the boundary, i.e. $\chi_i = \chi_r$



phase space
- position: ϕ_n
- direction of propagation: $p_n = \sin\chi_n$

the trajectories are curved in an inhomogeneous medium in analogy with the motion in a potentiel.

II. The Inhomogeneous Billiard Dynamics
The equations of motion

$$\frac{d}{ds}(n\vec{u}) = \nabla n \implies \begin{cases} \dfrac{du_r}{ds} = \dfrac{1}{n}\left[\dfrac{\partial n}{\partial r}u_\phi^2 - \dfrac{1}{r}\dfrac{\partial n}{\partial \phi}u_r u_\phi\right] + \dfrac{u_\phi^2}{r} \\ \dfrac{du_\phi}{ds} = \dfrac{1}{n}\left[\dfrac{1}{r}\dfrac{\partial n}{\partial \phi}u_r^2 - \dfrac{\partial n}{\partial r}u_\phi u_r\right] - \dfrac{u_\phi u_r}{r} \\ \dfrac{dr}{ds} = u_r \\ \dfrac{d\phi}{ds} = \dfrac{1}{r}u_\phi \end{cases}$$

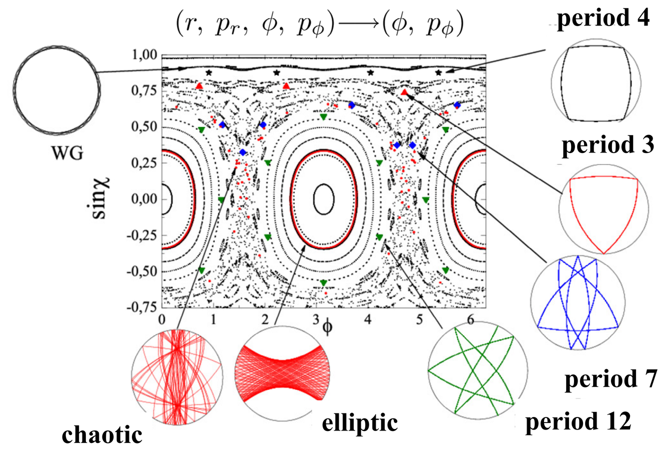
with a 5 parameter gaussian index of refraction:

$$n(r,\phi) = n_0 + \delta n e^{-2[(x-x_0)^2/w_x^2] - 2[(y-y_0)^2/w_y^2]}$$

inhomogeneity: $(\delta n/n_0, w_x)$ asymmetry: $(\epsilon = w_y/w_x, x_0, y_0)$

II. The Inhomogeneous Billiard Dynamics
Typical trajectories

$(r,\ p_r,\ \phi,\ p_\phi) \longrightarrow (\phi,\ p_\phi)$



$$(\phi_n, p_n) \xrightarrow{T} (\phi_{n+1}, p_{n+1}) \Longrightarrow \begin{cases} \phi_{n+1} = f(\phi_n,\ p_n) \\ p_{n+1} = g(\phi_n,\ p_n) \end{cases}$$

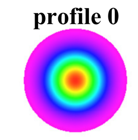
III. Regularity and Chaos in a Circular Cavity
Conditions of integrability and different profiles

integrability only when no angular dependence

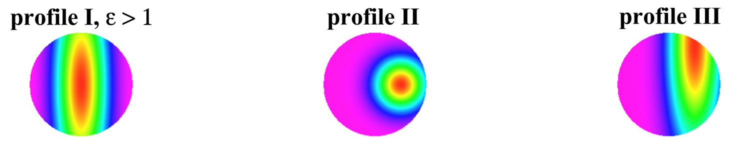
$$n(r) = n_0 + \delta n\, e^{-2r^2/w^2}$$

	w_x	w_y	x_0	y_0	integrable?
profile 0	w	w	0	0	yes
profile I	w	ϵw	0	0	no
profile II	w	w	$\neq 0$	0	no
profile III	w	ϵw	$\neq 0$	$\neq 0$	no

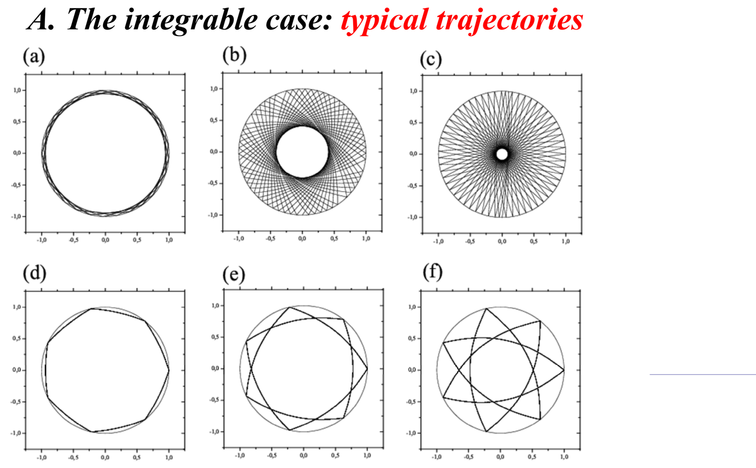
profile 0



profile I, ε > 1 — profile II — profile III



III. Regularity and Chaos in a Circular Cavity
A. The integrable case: typical trajectories

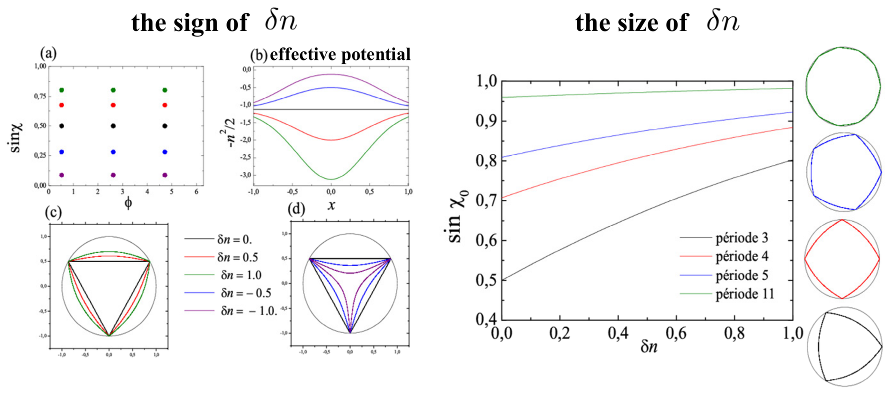


conservation of angular momentum: $\vec{L} = \vec{r} \times \vec{p} = R_0\, p\, \sin(\pi - \chi)\, \vec{e}_z$

analytical result: $u_\phi(r) = \dfrac{R_0\, n(R_0)}{r\, n(r)} \sin\chi_0$ radius of caustics: $r_c\, n(r_c) = R_0\, n(R_0) \sin\chi_0$

III. Regularity and Chaos in a Circular Cavity
A. The integrable case: effect of the inhomogeneity

the sign of δn — the size of δn



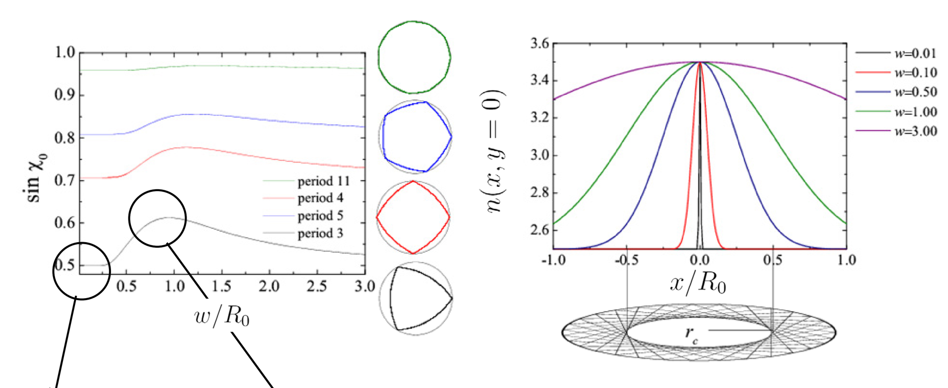
monotonic increase (decrease) of incident angle for δn > 0 (δn < 0)

effective potential is attractive (repulsive) for δn > 0 (δn < 0)

largest relative effect on trajectories of smaller periods

III. Regularity and Chaos in a Circular Cavity
A. The integrable case: effect of the inhomogeneity

the size of the width **w**



minimal effect until $w \sim r_c$

maximal effect when $w \sim \sqrt{2} r_c$

III. Regularity and Chaos in a Circular Cavity
B. The non-integrable case

the KAM scenario recovered (width asymmetry)

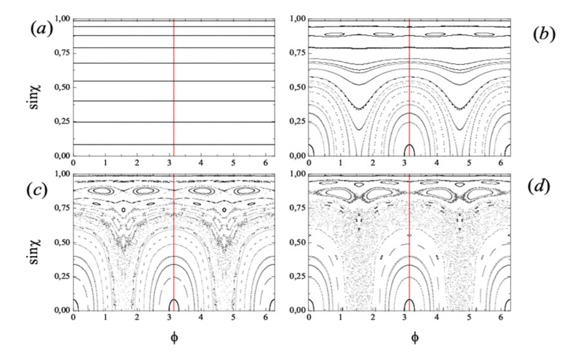
- the horizontal lines of constant sin χ are deformed
- destruction of regular structures (islands)
- birth of new pairs of elliptic and hyperbolic points

profile I

$(n_0 = 1.5, \delta n = 1, w = R_0)$

a) $\epsilon = 0.0$ b) $\epsilon = 1.25$

c) $\epsilon = 2$ d) $\epsilon = 8$



III. Regularity and Chaos in a Circular Cavity
B. The non-integrable case

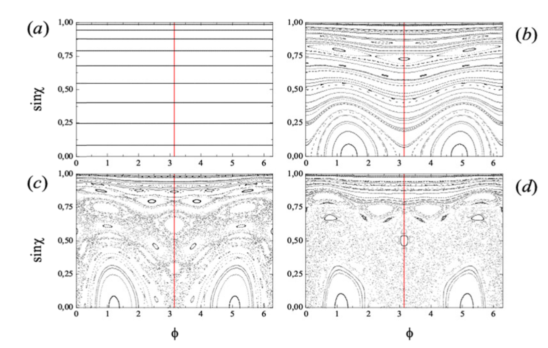
the KAM scenario recovered (displacement from the center)

profile II

$(n_0 = 1.5, \delta n = 1, w = R_0)$

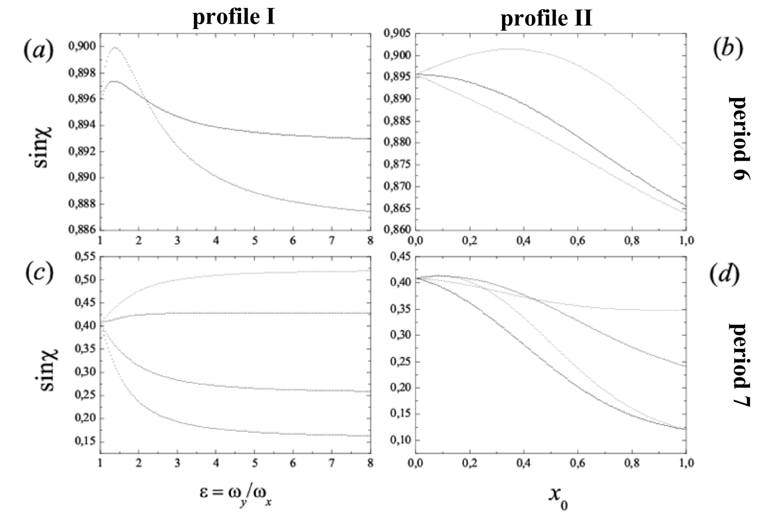
a) $x_0 = 0.0$ b) $x_0 = 0.25\, R_0$

c) $x_0 = 0.5\, R_0$ d) $x_0 = 0.75\, R_0$

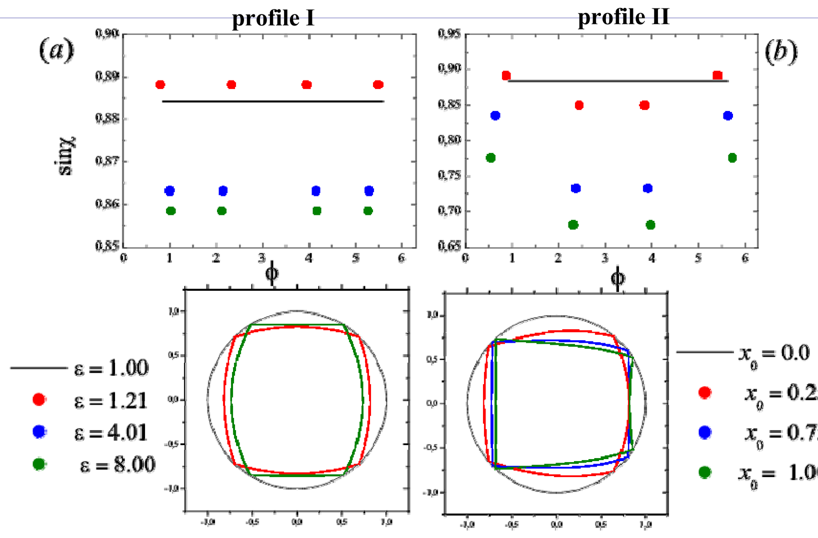


III. Regularity and Chaos in a Circular Cavity
B. The non-integrable case: effect on the incident angles

$(n_0 = 1.5, \delta n = 1, w = R_0)$



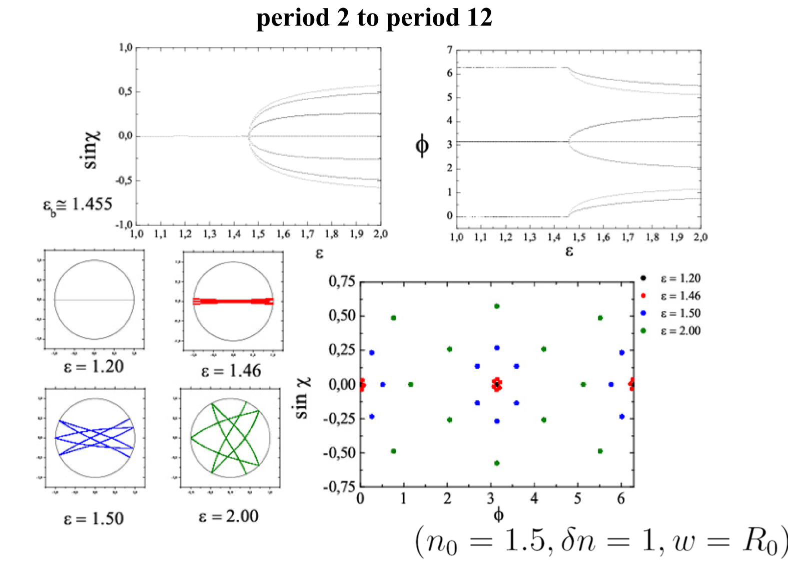
III. Regularity and Chaos in a Circular Cavity
B. The non-integrable case: effect on a period 4



$(n_0 = 1.5, \delta n = 1, w = R_0)$

III. Regularity and Chaos in a Circular Cavity
B. The non-integrable case: inducing bifurcations

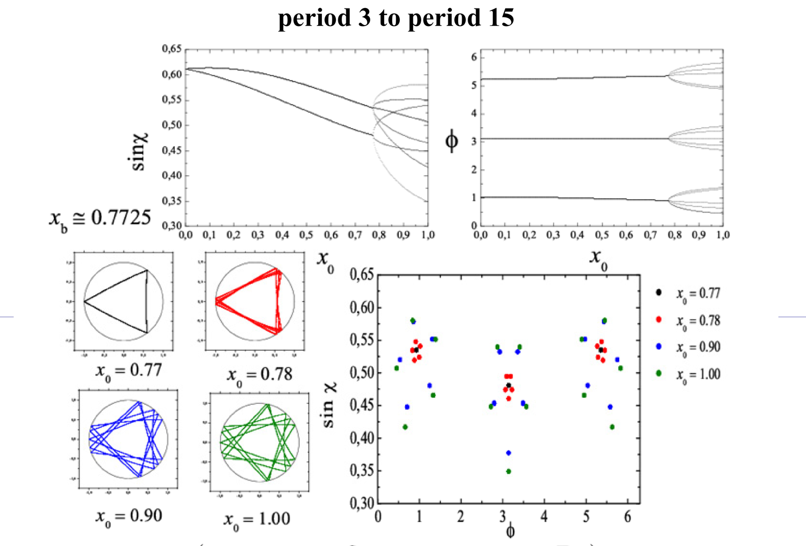
period 2 to period 12



$(n_0 = 1.5, \delta n = 1, w = R_0)$

III. Regularity and Chaos in a Circular Cavity
B. The non-integrable case: inducing bifurcations

period 3 to period 15



$(n_0 = 2.5, \delta n = 0.5, w = R_0)$

III. Regularity and Chaos in a Circular Cavity
B. The non-integrable case: increasing chaos

the stronger the inhomogeneity (δn and/or ε),

the stronger is *CHAOS*

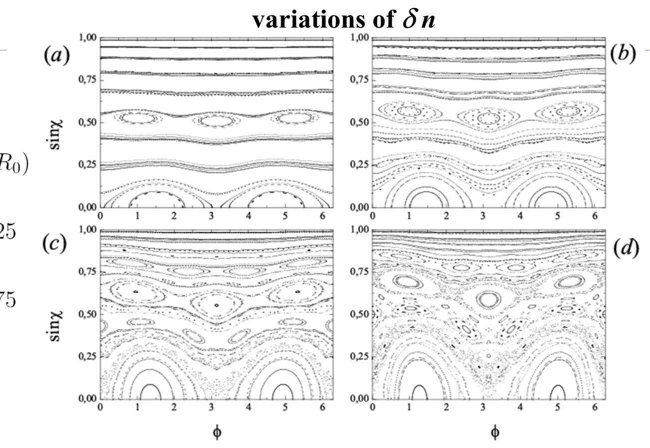
variations of δn

profile II

$(n_0 = 1.5, w = R_0, x_0 = 0.5\, R_0)$

a) $\delta n = 0.1$ b) $\delta n = 0.25$

c) $\delta n = 0.5$ d) $\delta n = 0.75$

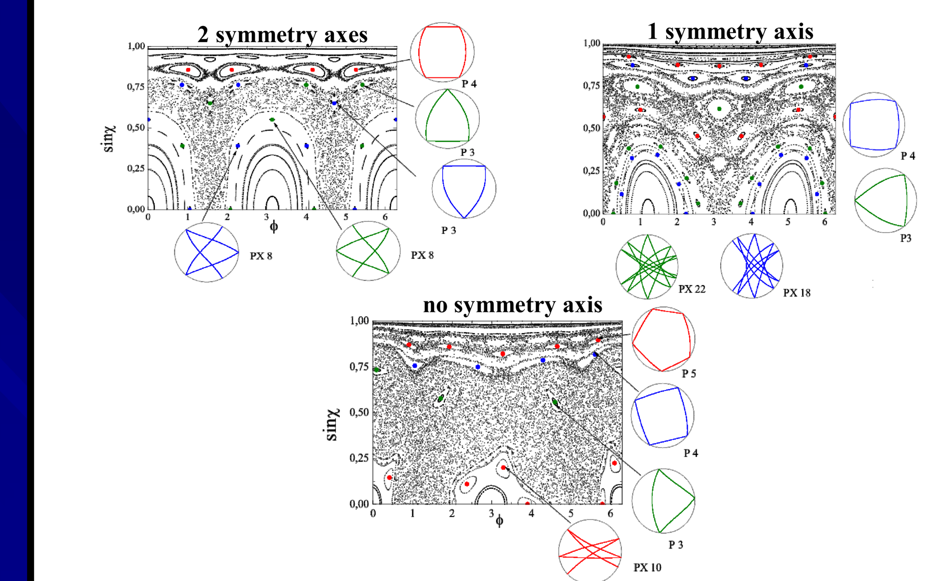


IV. Symmetry and Degeneracy in an Inhomogeneous Circular Cavity
Classification

symmetry of system	symmetry of trajectory	degeneracy
rotation	arbitrary	∞
Ox and Oy	Ox and Oy	1
Ox and Oy	Ox or Oy	2
Ox and Oy	none	4
Ox or Oy	Ox or Oy	1
Ox or Oy	Oy or Ox	2
Ox or Oy	none	2
none	arbitrary	1

degeneracy:
to one periodic orbit correspond many different trajectories
many sin χ components of the same trajectory are identical

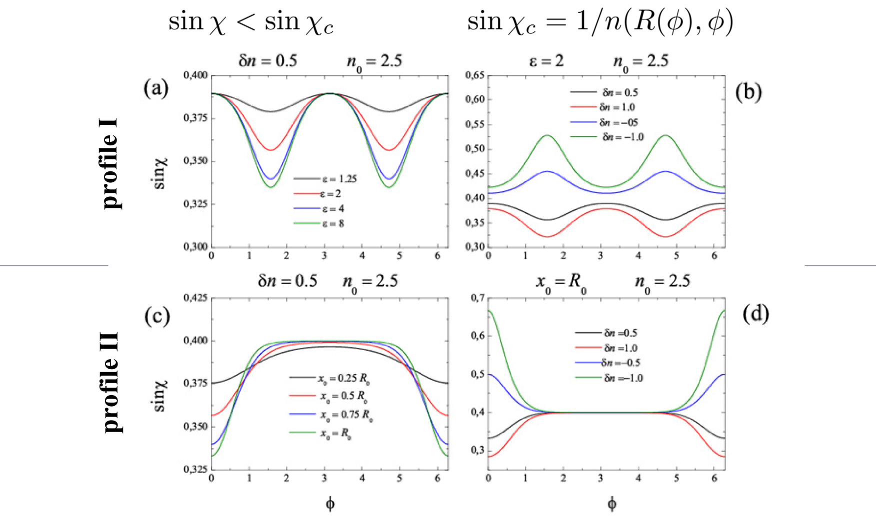
IV. Symmetry and Degeneracy in an Inhomogeneous Circular Cavity
Some examples



V. Refractive Escape in an Inhomogeneous Circular Billiard

the condition for refractive escape is no longer a straight line, but is determined by the condition

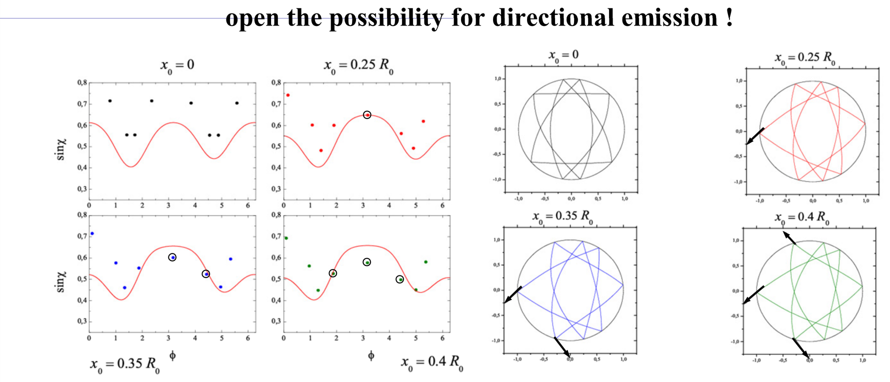
$\sin\chi < \sin\chi_c$ $\sin\chi_c = 1/n(R(\phi), \phi)$



V. Refractive Escape in an Inhomogeneous Circular Billiard

displacement of the trajectories
and
deformation of
the line of refractive escape

open the possibility for directional emission !



VI. Summary and Conclusions

- introduction of a new class of billiards, the *inhomogeneous billiard*
- transition from regular to mixed phase space (chaos) *in an integrable billiard geometry* (here circle)
- perturbation (*symmetry breaking*) is achieved by a 5 parameter gaussian index of refraction (2 inhomogeneity and 3 asymmetry parameters)
- demonstration of our ability to influence (control) the dynamical features and behaviours

- in the context of microresonators (*microlasers*), the classical ray dynamics in a *photonic billiard* is a useful tool to understand the emission properties of the cavity
- our new approach offers the possibility to affect the *refractive escape* properties of the cavity and to shape *the unstable manifolds* responsible for the directionality of the emission