NetSci2020

Influence maximization in simplicial contagion

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ARTICLE

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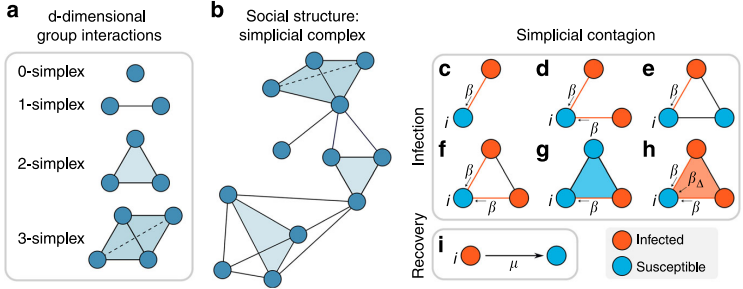
Simplicial models of social contagion

Iacopo Iacopini [1,2], Giovanni Petri[3,4], Alain Barrat [3,5] & Vito Latora [1,2,6,7]

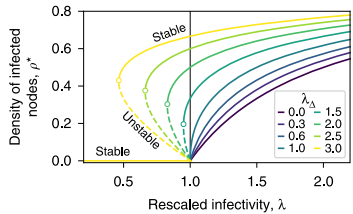


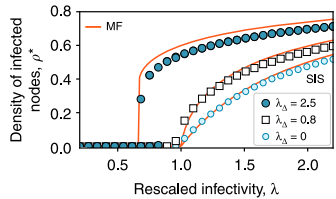
Mean-field description

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -I + \sum_{w} \beta_\omega \langle k_\omega \rangle I^\omega (1 - I) \ .$$

- $I(t)$: fraction of infected nodes
- $\langle k_\omega \rangle$: average participation to ω-simplex
- β_ω : additive infection rate when ω nodes are infected within a simplex

Not appropriate for heterogeneous structures!





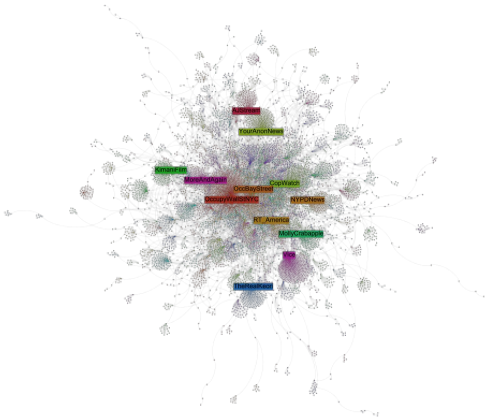
Some related works

- N. Landry, J. G. Restrepo : *The effect of heterogeneity on hypergraph contagion models*
- B. Jhun, M. Jo and B. Kahng : *Simplicial SIS model in scale-free uniform hypergraph*
- J. T. Matamalas, S. Gómez, A. Arenas : *Abrupt phase transition of epidemic spreading in simplicial complexes*
- P. Cisneros-Velarde, F. Bullo : *Multi-group SIS epidemics with simplicial and higher-order interactions*

Key points of the talk

1. An analytical approach to contagions on higher-order networks
2. Dynamical heterogeneity of groups/simplices
3. "Influential groups/simplices" can beat "influential spreaders"

Who influences Twitter discussions? #myNYPD



Graphical representation of the node-link structure of the #myNYPD retweet/mention network.

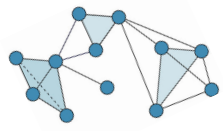
1. No correlation between # of followers and influence (retweets+mentions), $r = 0.145$.
2. Clashes with standard notions of "influential spreaders".

S. Jackson & B. Foucault Welles, Journal of Communication, 2015

Who are the INFLUENTIAL SPREADERS *of* COMPLEX CONTAGIONS *on networks with* HIGHER-ORDER STRUCTURE*?*

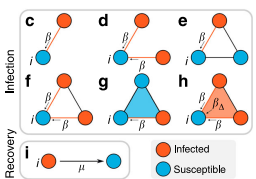
Mapping simplagion to complex contagion on bipartite networks

Simplicial complex



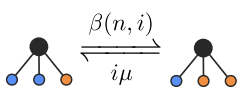
Bipartite network

Simplicial contagion



Group-based contagion

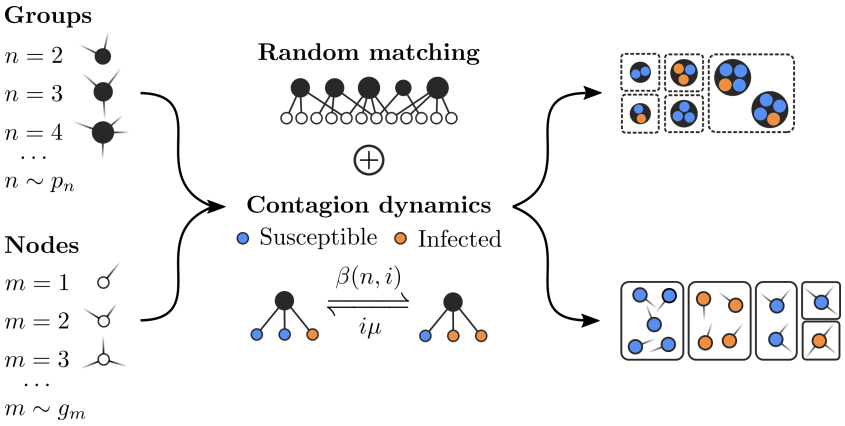
Susceptible Infected



n : Group size

$i \leq n$: # of infected

Higher-order analytical framework



Heterogeneous mean-field equations for nodes

$$\frac{\mathrm{d}s_m}{\mathrm{d}t} = 1 - s_m - m\, r\, s_m \ .$$

Approximate master equations for groups

$$\begin{aligned}\frac{\mathrm{d}f_{n,i}}{\mathrm{d}t} = & \ \mu(i+1)\, f_{n,i+1} - \mu i\, f_{n,i} \ , \\ & - (n-i)\left[\beta(n,i) + \rho\right] f_{n,i} \ , \\ & + (n-i+1)\left[\beta(n,i-1) + \rho\right] f_{n,i-1} \ .\end{aligned}$$

Example

$f_{3,2}$ 

s_3 

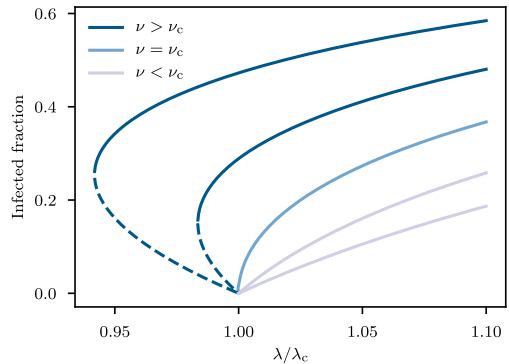
LHD et al. Phys Rev E, 2010

- $s_m(t)$: fraction of susceptible nodes with membership m
- $f_{n,i}(t)$: fraction of groups of size n with i infected
- $\beta(n,i)$, μi : local infection/recovery rates
- $r(t)$, $\rho(t)$: mean-field couplings

Simple model of social contagion

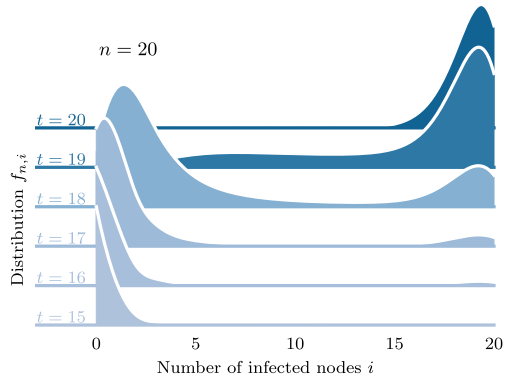
$$\beta(n, i) = \lambda i^{\nu}$$

- $\nu < 1$: social inhibition
- $\nu = 1$: SIS model
- $\nu > 1$: social reinforcement



Dynamical heterogeneity of groups

- Groups of the same size do not all follow the same evolution.
- Bimodality of outcomes would be lost in a coarse-grained model.
- Can we maximize the faster mode?



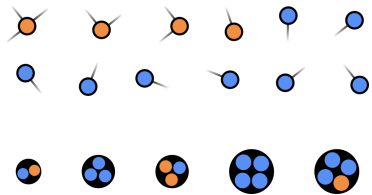
Goal : *Maximize* $\dot{I}(0)$ *by distributing wisely* $I(0) = \epsilon \ll 1$.

Rules

- We set $\lambda > \lambda_c$ so that $I^* = 0$ is unstable
- You can choose among two approaches
  1. *Influential spreaders* : engineer node set $\{s_m(0)\}$
  2. *Influential simplices* : engineer group set $\{f_{n,i}(0)\}$
- The unchosen set is distributed randomly, i.e.

$$f_{n,i}(0) = \binom{n}{i} \epsilon^i (1-\epsilon)^{n-i} \quad \text{or} \quad s_m = 1 - \epsilon \; \forall m \; .$$

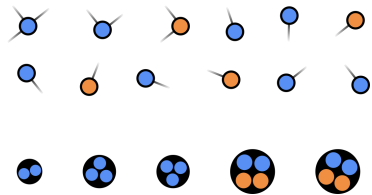
Influential spreaders



Optimal strategy

Infect nodes with highest available membership m

Influential groups

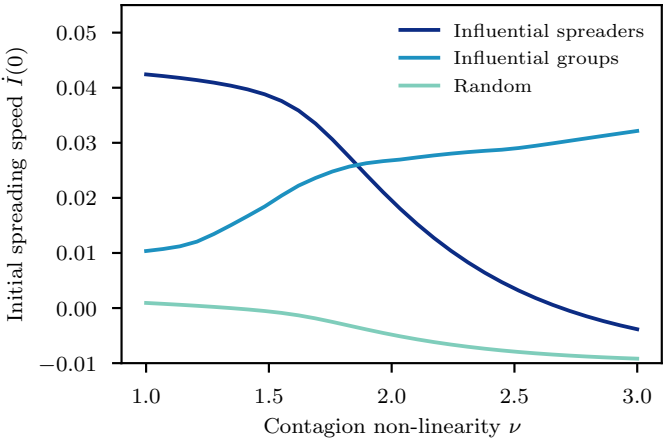


Optimal strategy

Favor most *profitable* group configurations (n, i) as measured from $R(n, i) = \beta(n, i)(n - i)/i$

Influential groups beat influential spreaders in strongly non-linear contagions

$$g_m \sim m^{-\gamma_m} \; ; \; p_n \sim \theta^n e^{-\theta}/n!$$



What's next?

- The classic picture of influential spreaders sometimes fail. But when?
- Understand when to target INFLUENTIAL GROUPS *or* influential spreaders.
- Look at the reverse problem : targeted immunization.
  - Is it better to immunize nodes or parts of groups?
- ...

Take-home message

1. We have models to help us think more deeply about the interplay of higher-order structure and non-linear contagions

2. These models shift the focus from individuals to groups

3. Influential groups/simplices vs influential spreaders/nodes

Aknowledgments

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Preprints using the same framework

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