Time-dependent connection threshold in growing random geometric graphs

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Random geometric graphs (RGG) provide a realistic and compact modeling approach to complex networks. In this class of models, nodes are assigned coordinates in a metric space and are connected if they are separated by a shorter distance than a given threshold R [1]. The specifics of the embedding space can then be used to characterize and reproduce a wide range of structural features, such as high clustering coefficients and scale-free degree distributions.

We propose a new model of growing RGGs in which the threshold distance for connections is a general, arbitrary time-dependent function, $R(t)$. We provide a complete mathematical analysis which yields analytical expressions for the degree distribution, $P(k)$, and the average clustering coefficient, $\langle c \rangle$, among others. We also establish the correspondence between our model and the hyperbolic model of Ref. [2]. Furthermore, we propose an efficient algorithm to infer $R(t)$ for real growing complex networks using the temporal ordering of the nodes and their degree. Finally, we investigate the effect of various general forms of $R(t)$, and show how an appropriate choice for $R(t)$ mimics preferential attachment without an explicit knowledge of the degree of nodes.

[1] J. Dall and M. Christensen, "Random Geometric Graphs" Phys. Rev. E **66**, 016121 (2002).
[2] D. Krioukov, F. Papadopoulos, M. Kitsak, A. Vahdat and M. Boguñá, "Hyperbolic geometric of complex networks", Phys. Rev. E **82**, 036106 (2010).

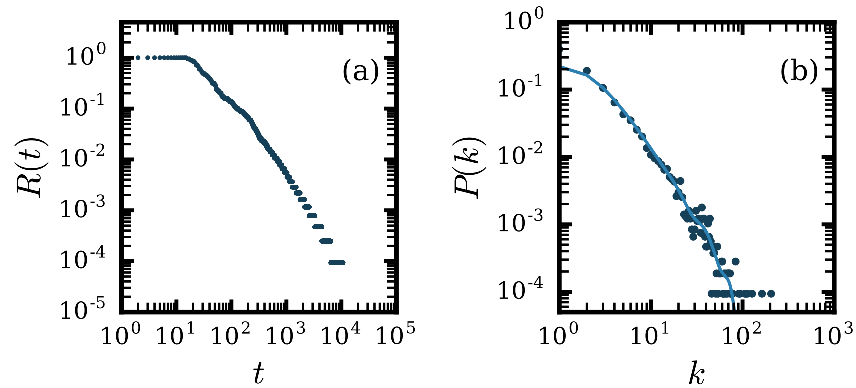

Figure 1: Inference on the Pretty Good Privacy (PGP) network. (a) Inferred connection threshold, R(t), as a function of time, t. The degree of nodes has been used as a proxy of their age. (b) Degree distribution where markers correspond to the empirical data and the line shows the analytical prediction of our model using $R(t)$ given in (a).